Hyperpropreties

Hadar Frenkel, January 13, 2025

Introduction to Temporal Hyperpropreties

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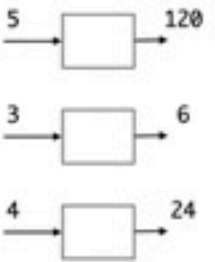
Motivation

Is Our Program Correct?

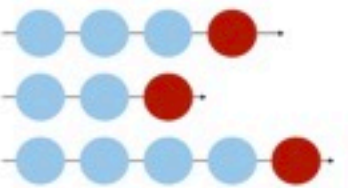
Is Our Program Correct?

Trace Properties
allowed behaviours

Functional correctness



Termination



Correctness is not Enough

Systems can be correct but still not:

Secure Fair Robust

...

Correctness is not Enough

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Secure Fair Robust

...

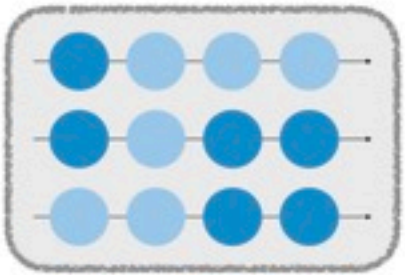
What is the set of properties the system should satisfy?

Hyperproperties

System properties

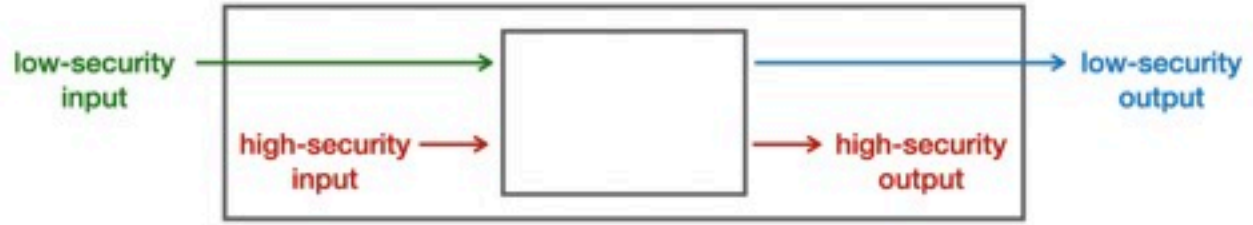
Relate multiple executions

Information-flow security



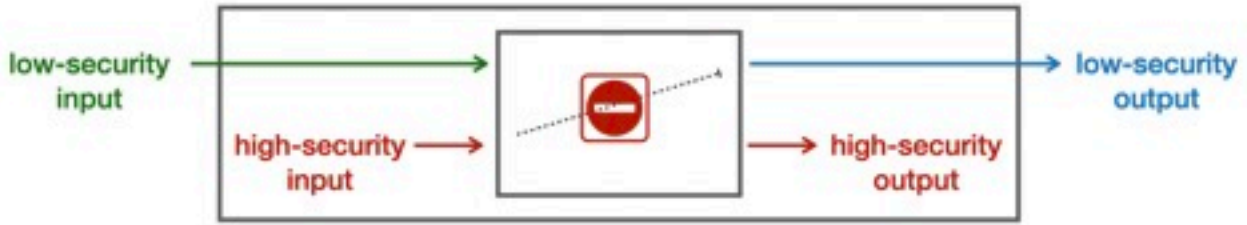
Hyperproperties. Clarkson and Schneider (CSF 2008)

Example: Noninterference



Security Policies and Security Models. Goguen and Meseguer (IEEE 1982)

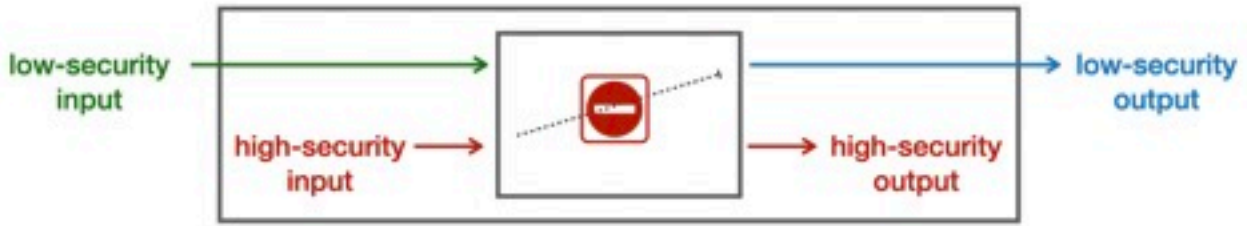
Example: Noninterference



Low-security output can only depend on low-security input

Security Policies and Security Models. Goguen and Meseguer (IEEE 1982)

Example: Noninterference



Low-security output can only depend on low-security input

For every two executions:
if they agree on low-security input then
they agree on low-security output

Security Policies and Security Models. Goguen and Meseguer (IEEE 1982)

Hyperproperties

System properties

Relate multiple executions

Information-flow

Hyperproperties

System properties

Relate multiple executions

Information-flow

Knowledge

Hyperproperties

System properties

Relate multiple executions

Information-flow

Knowledge

Fairness

Hyperproperties

System properties

Relate multiple executions

Information-flow

Knowledge

Fairness

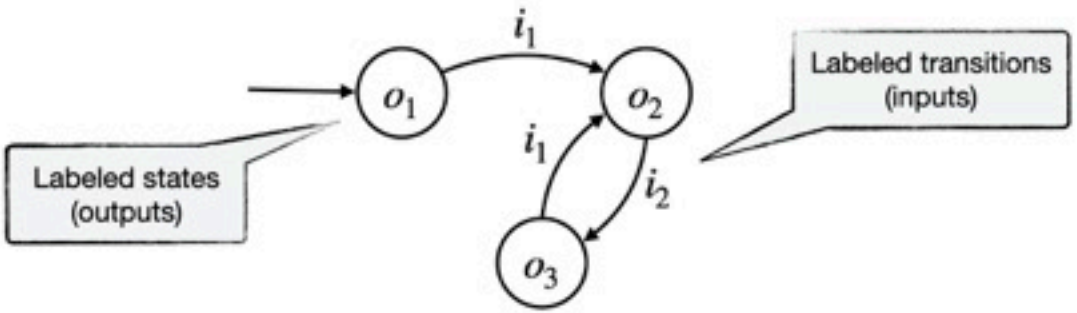
Robustness

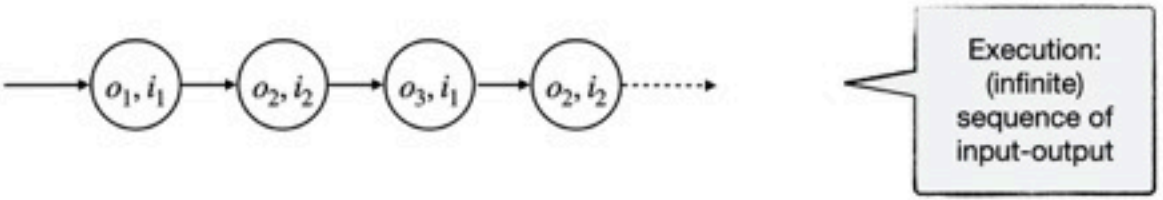
Hyperproperties are important!
Let’s verify them

Outline

- Trace properties & Hyperproperties
- HyperLTL - a logic for temporal hyperproperties
- Some interesting properties in HyperLTL
- HyperLTL model-checking
- HyperLTL satisfiability
- Beyond HyperLTL and security

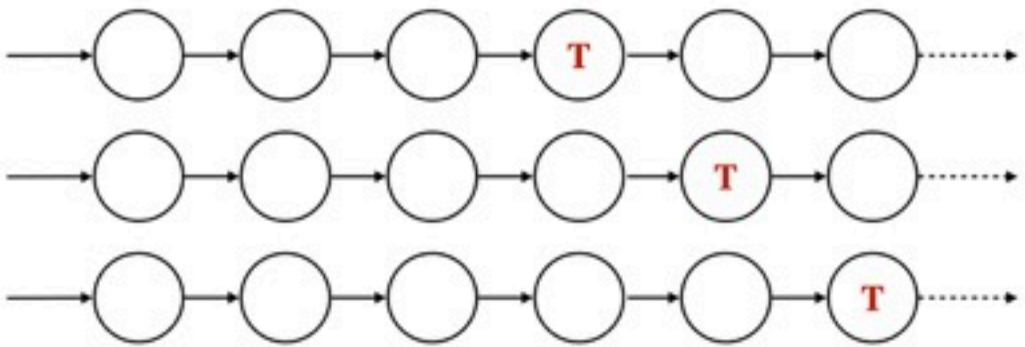
Setting


i_1
o_1
o_2
i_1
i_2
o_3
Labeled states
(outputs)
Labeled transitions
(inputs)


o_1, i_1
o_2, i_2
o_3, i_1
o_2, i_2
Execution:
(infinite)
sequence of
input-output

Trace Properties

"All executions reach T"



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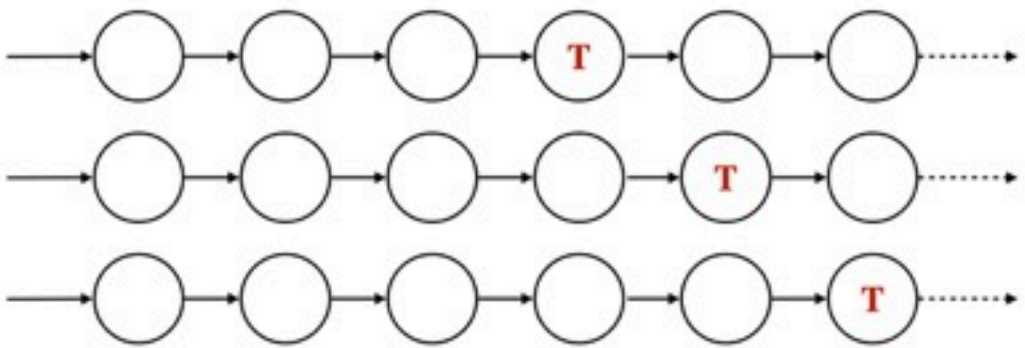

We can look at each trace separately and verify the property

Trace Properties

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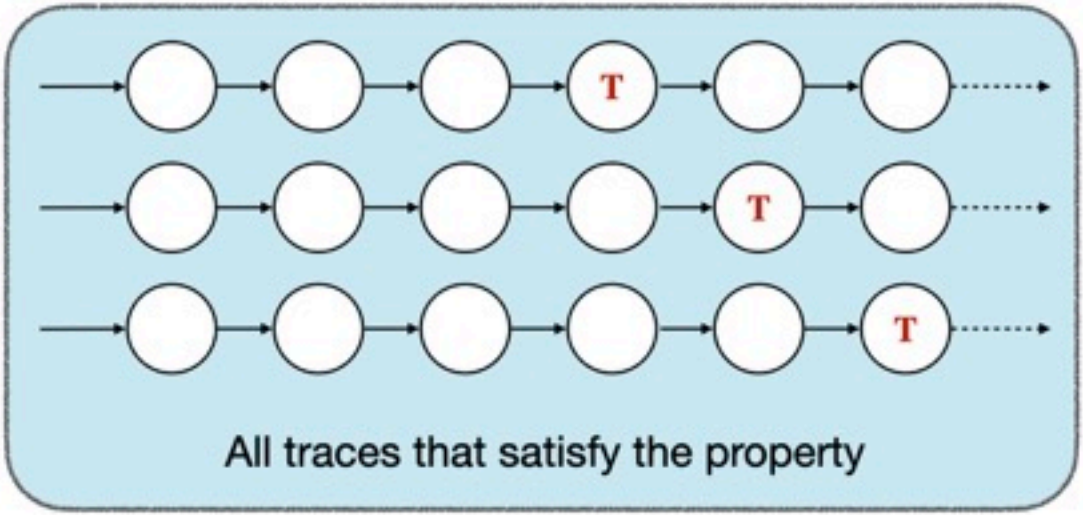

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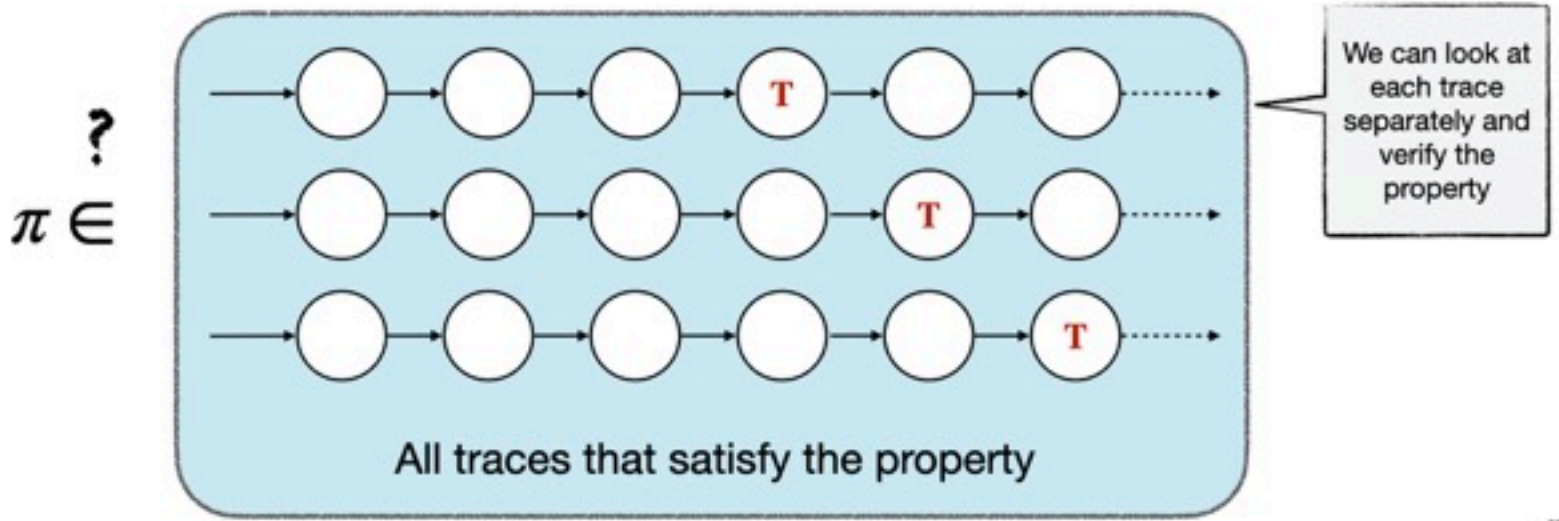

Hyperproperties

"All executions reach T at the same time"

We have to compare different executions

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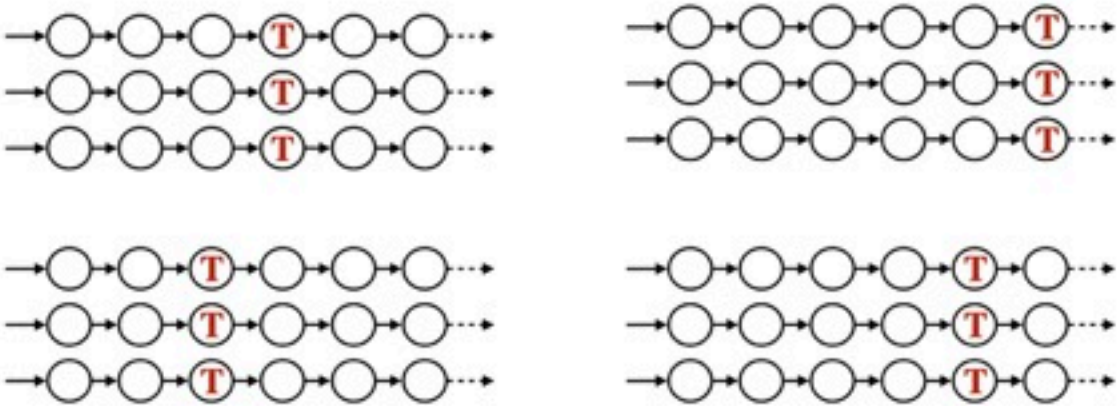
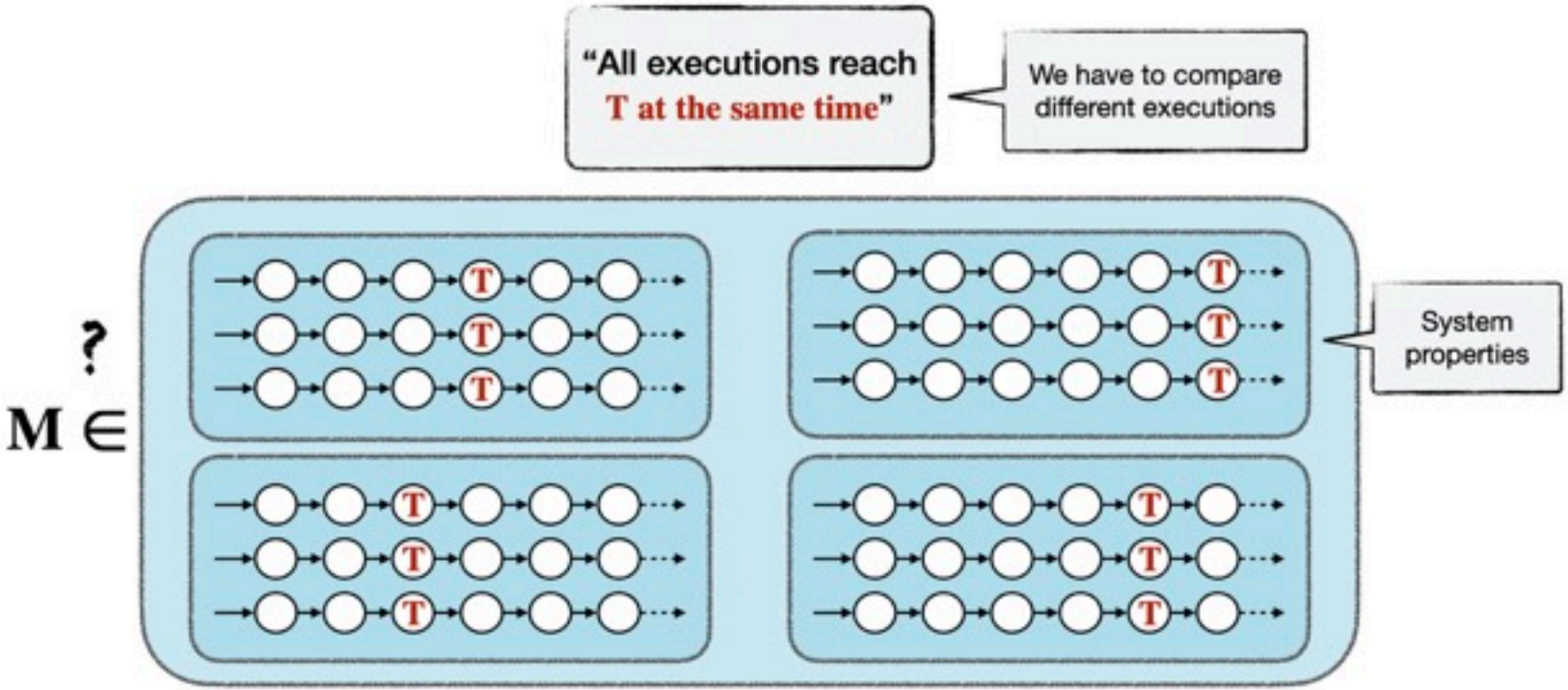
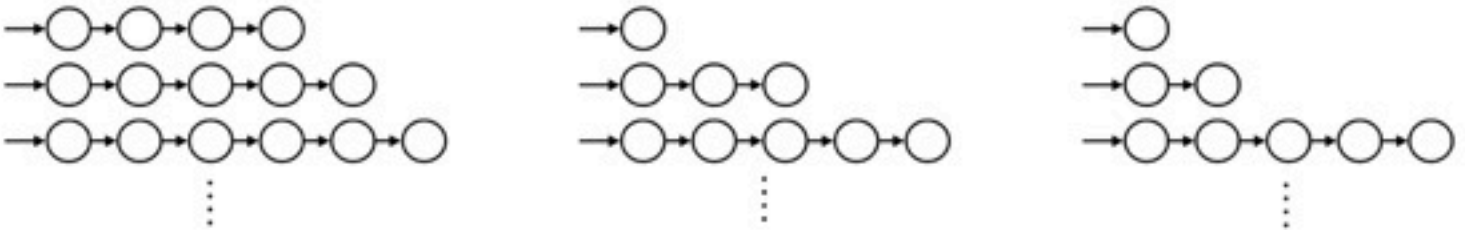


Hyperproperties



Hyperproperties, Example 2

All infinite languages



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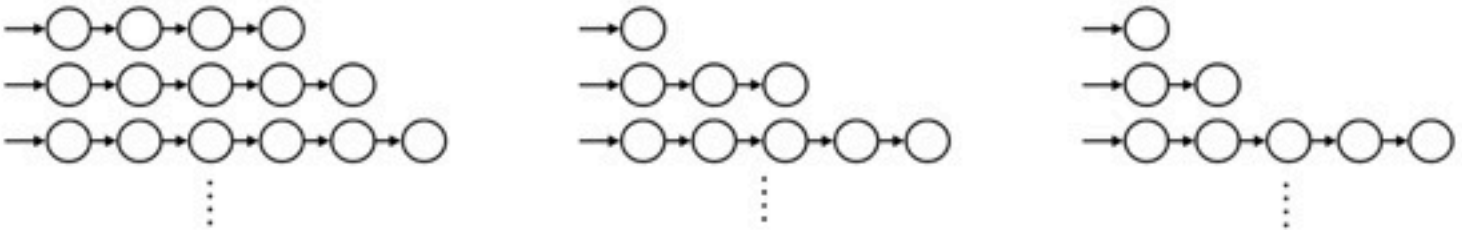
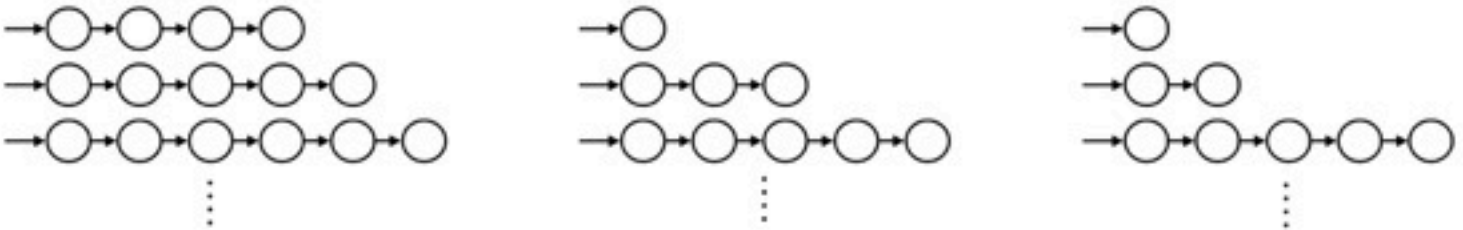


for every trace
exists a longer trace

Hyperproperties, Example 2

All infinite languages



for every trace
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&
exists at least one trace

Hyperproperties, Example 2

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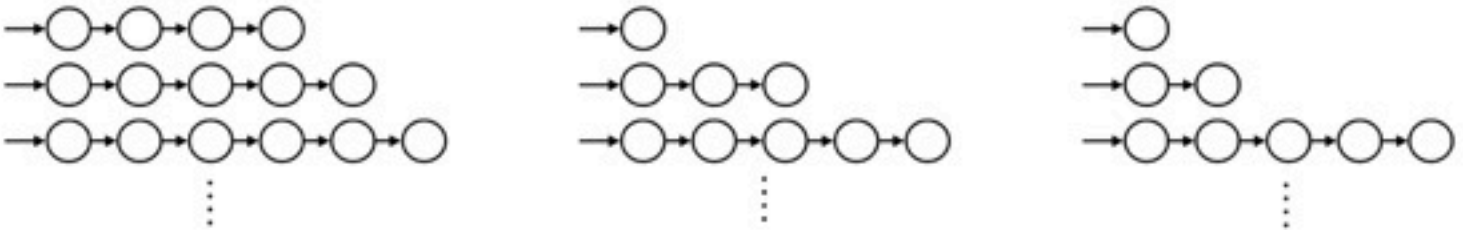

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exists at least one trace

Quantification over traces!

LTL

$$\psi := a \mid \neg\psi \mid \psi \wedge \psi \mid \bigcirc \psi \mid \psi \cup \psi'$$

ψ holds at the next step

ψ holds until ψ' holds

temporal operators
describe behavior through time:
$\Diamond\psi$ – ψ eventually holds
$\Box\psi$ – ψ holds all the time (globally)

The Temporal Logic of Programs. Pnueli (FOCS 1977).

HyperLTL

$$\psi := a_\pi \mid \neg\psi \mid \psi \wedge \psi \mid \bigcirc \psi \mid \psi \mathbin{U} \psi'$$

$$\varphi := \exists \pi . \varphi \mid \forall \pi . \varphi \mid \psi$$

trace variable

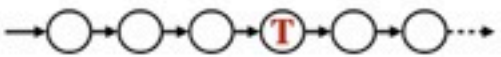
Temporal Logics for Hyperproperties. Clarkson, Finkbeiner, Koleini, Micinski, Rabe, and Sánchez. POST14.

HyperLTL Example

"All executions reach T at the same time"

$$\exists \pi . \neg T_\pi \; U \; (T_\pi \wedge \bigcirc \Box \neg T_\pi)$$

T holds exactly once on π



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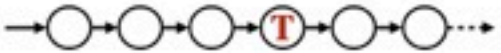
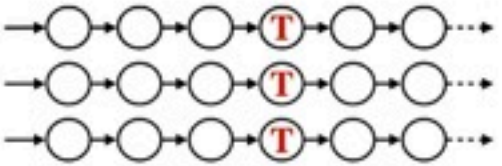


For all other traces π', T holds at the same time as in π



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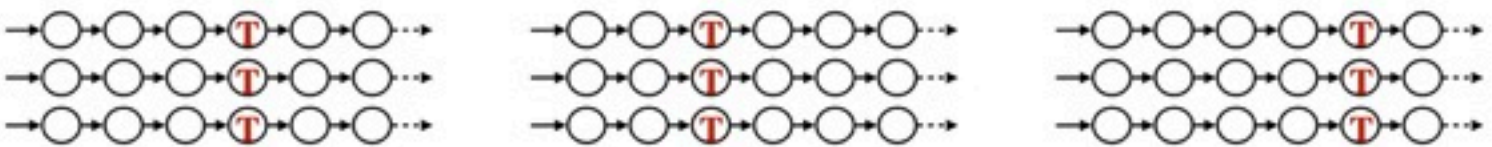
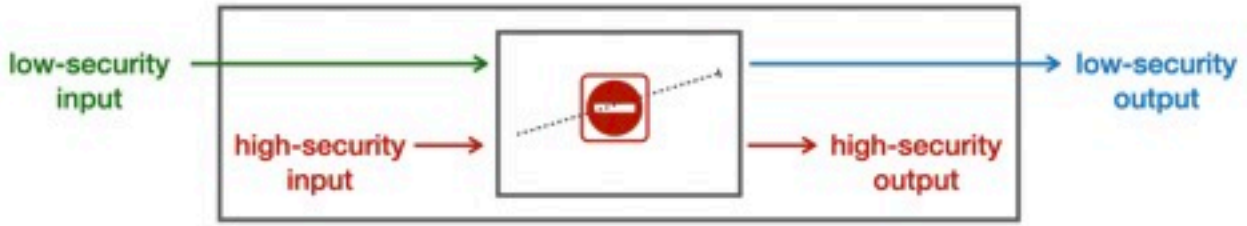


Example: Noninterference



Low-security output can only depend on low-security input

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Security Policies and Security Models. Goguen and Meseguer (IEEE 1982)

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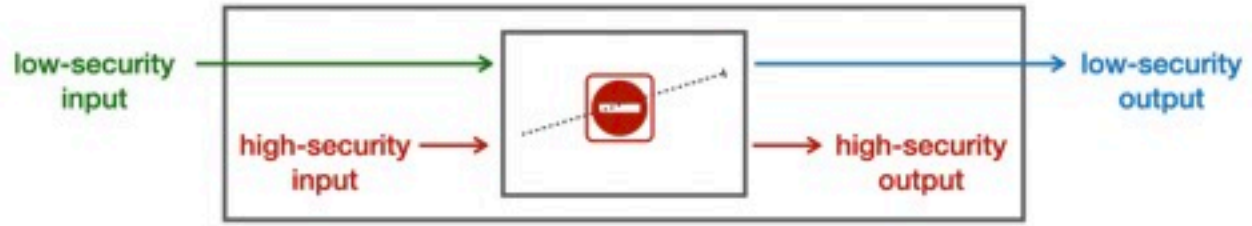

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$$\forall \pi \forall \pi' . \Box(low - in_{\pi} \leftrightarrow low - in_{\pi'}) \rightarrow \Box(low - out_{\pi} \leftrightarrow low - out_{\pi'})$$

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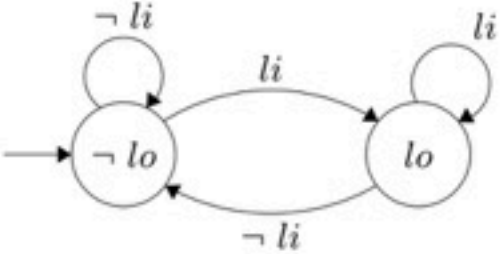

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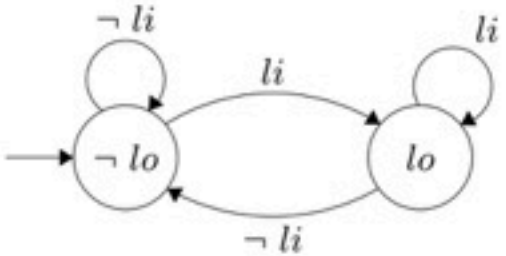

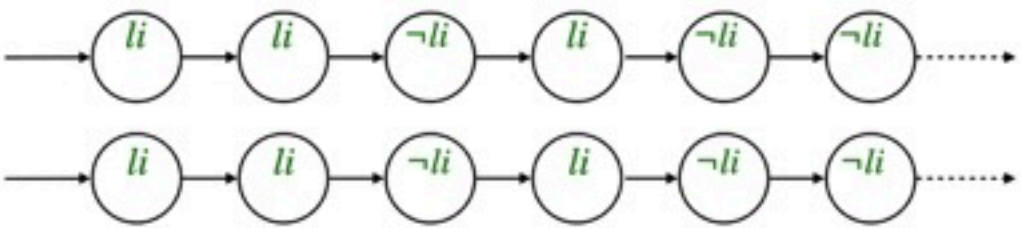

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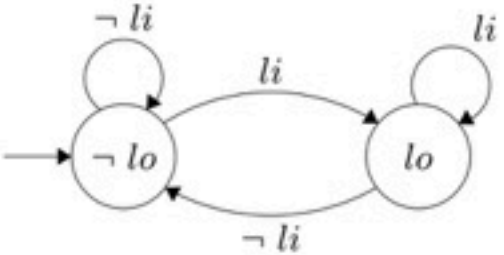

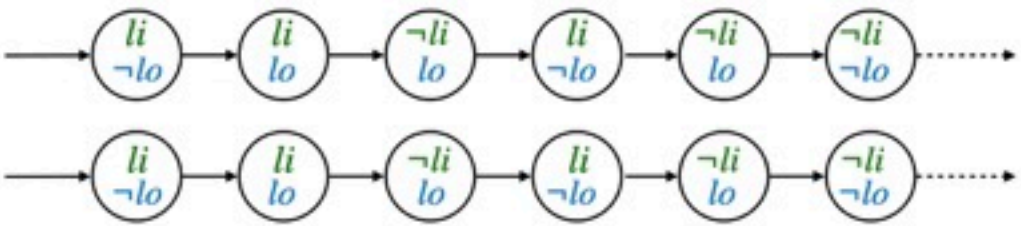

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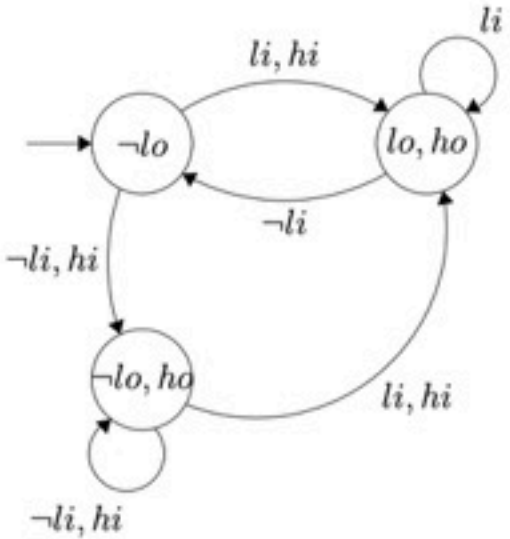

Systems can be more complex and include high-security variables

But our examples will remain simple

Outline

- Trace properties & Hyperproperties
- HyperLTL - a logic for temporal hyperproperties
- Some interesting properties in HyperLTL
- HyperLTL model-checking
- HyperLTL satisfiability
- Beyond HyperLTL and security

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- Trace properties & Hyperproperties
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  - $\exists^*$-HyperLTL
  - $\forall^*$-HyperLTL
  - Quantifier alternation
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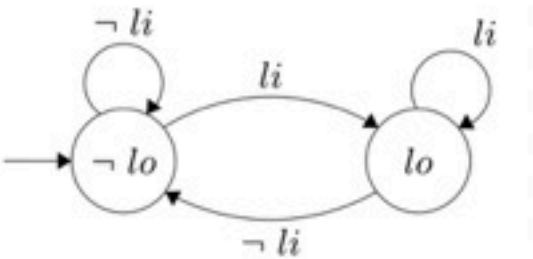
Model-Checking ∃*HyperLTL

$$\exists \pi \exists \pi' . \square (low - in_{\pi} \leftrightarrow low - in_{\pi'})$$

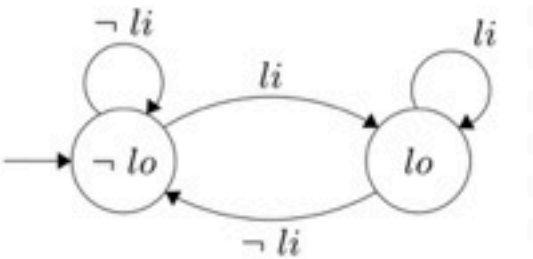


*Algorithms for Model Checking HyperLTL and HyperCTL**. Finkbeiner, Rabe, and Sánchez. CAV 2015.

Model-Checking ∃*HyperLTL

$$\exists \pi \exists \pi' . \square (low - in_{\pi} \leftrightarrow low - in_{\pi'})$$

Look for witness traces



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Model-Checking $\exists^*$HyperLTL

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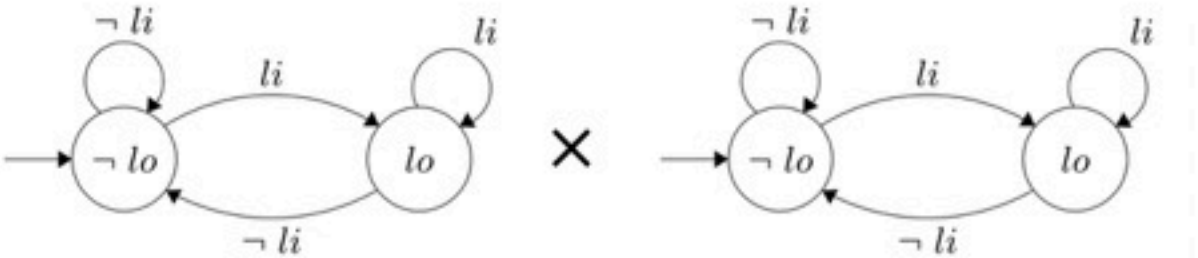
Look for witness traces in the composed system



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Model-Checking $\exists^*$HyperLTL

HyperLTL to an Automaton

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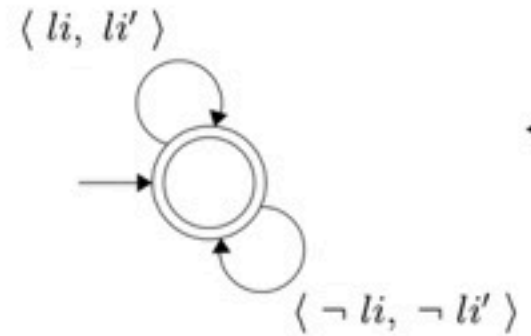

2 quantifiers, alphabet is $\Sigma \times \Sigma$
k quantifiers, alphabet is Σ^k

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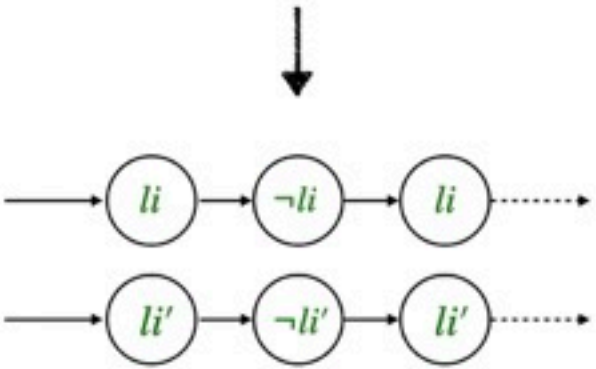
Model-Checking $\exists^*$HyperLTL

HyperLTL to an Automaton

$$\langle li, li' \rangle, \langle \neg li, \neg li' \rangle, \langle li, li' \rangle, \cdots$$



2 quantifiers, alphabet is $\Sigma \times \Sigma$
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Traces over k-tuples of letters represent k traces

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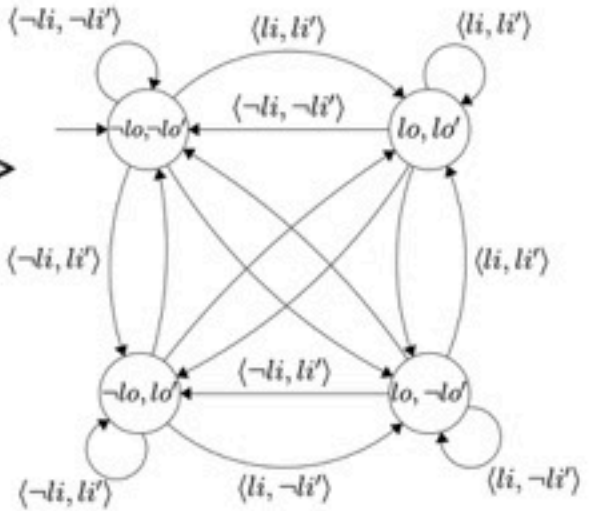
Model-Checking ∃*HyperLTL

$$\exists \pi \exists \pi' . \square (low - in_{\pi} \leftrightarrow low - in_{\pi'})$$

Self-compose the system (k times)



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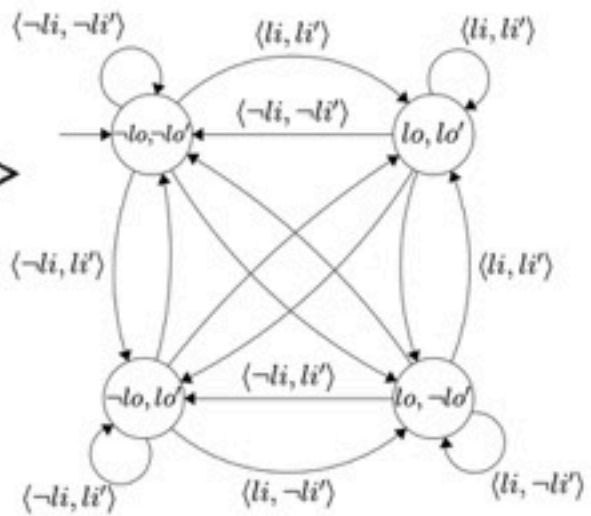

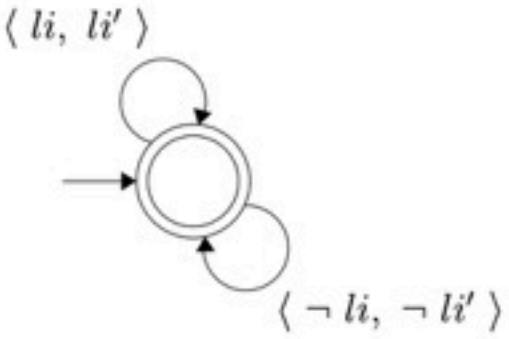

Intersect with the inner formula automaton, to find witness traces

Algorithms for Model Checking HyperLTL and HyperCTL.* Finkbeiner, Rabe, and Sánchez. CAV 2015.

Model-Checking $\exists^*$HyperLTL

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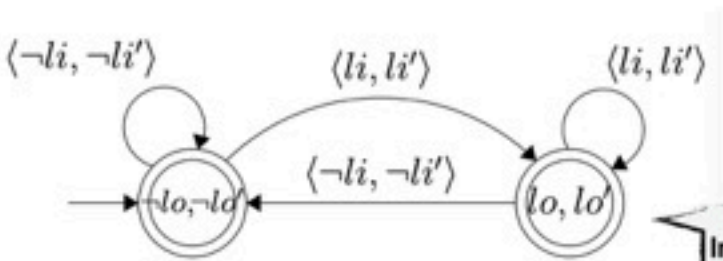

Intersection of the inner formula and the 2-slef composition of the system

Test for emptiness

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Model-Checking $\exists^*$HyperLTL

$$\exists \pi_1 \cdots \exists \pi_k \,.\, \psi$$

1. A_ψ — an automaton for the inner formula ψ, over k-tuples of letters
2. M_k — a k self-composition of the system
3. Intersect $A_\psi \cap M_k$ and check for emptiness
4. A k-trace in $A_\psi \cap M_k$ represents the witness traces $\pi_1, \cdots, \pi_k$

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Model-Checking $\forall^*$HyperLTL

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2 quantifiers, alphabet is $\Sigma \times \Sigma$

This time we are looking for **violation**

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Model-Checking ∀*HyperLTL

$$\forall \pi \forall \pi' . \square (low - in_{\pi} \leftrightarrow low - in_{\pi'})$$

2 quantifiers, alphabet is $\Sigma \times \Sigma$

$$\exists \pi \exists \pi' . \Diamond (low - in_{\pi} \leftrightarrow \neg low - in_{\pi'})$$

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This looks familiar…

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Model-Checking $\forall^*$HyperLTL

$$\forall \pi_1 \cdots \forall \pi_k \,.\, \psi$$

1. Negate the formula — $\exists \pi_1 \cdots \exists \pi_k \,.\, \neg\psi$
2. $A_{\neg\psi}$ — an automaton for the inner formula $\neg\psi$, over k-tuples of letters
3. M_k — a k self-composition of the system
4. Intersect $A_{\neg\psi} \cap M_k$ and check for emptiness
5. A k-trace in $A_{\neg\psi} \cap M_k$ represents the counterexample traces $\pi_1, \cdots, \pi_k$

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Outline

- Trace properties & Hyperproperties
- HyperLTL - a logic for temporal hyperproperties
- Some interesting properties in HyperLTL
- HyperLTL model-checking
  - $\exists^*$-HyperLTL
  - $\forall^*$-HyperLTL
  - Quantifier alternation
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Model-Checking $\forall\exists$HyperLTL

Generalized Noninterference

$$\forall\pi\forall\pi'\exists\pi''.\,\square(high-in_{\pi} \leftrightarrow high-in_{\pi''}) \wedge \square(low-out_{\pi'} \leftrightarrow low-out_{\pi''})$$

For every two traces
there exists a third trace
that agrees with one on the high-security inputs,
and with the other on the low-security outputs

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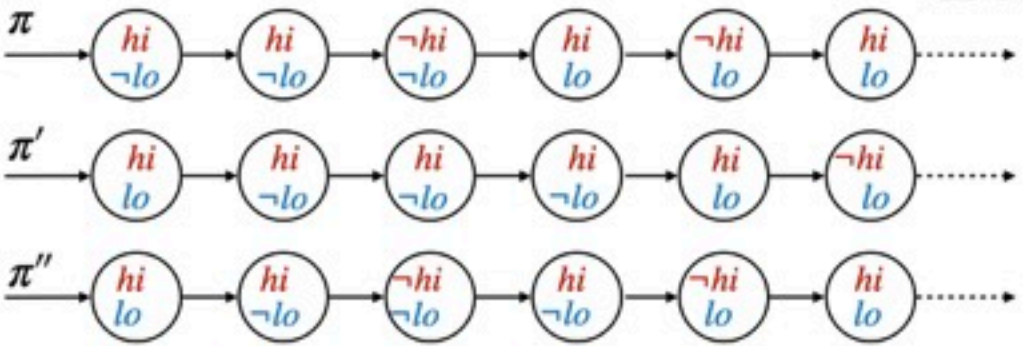

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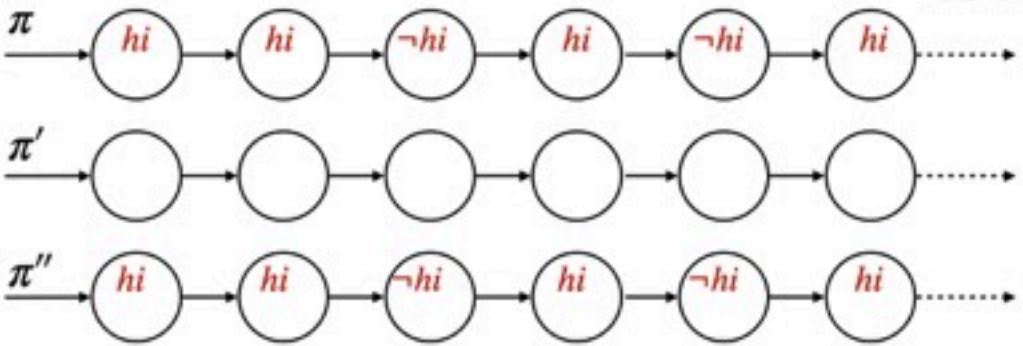

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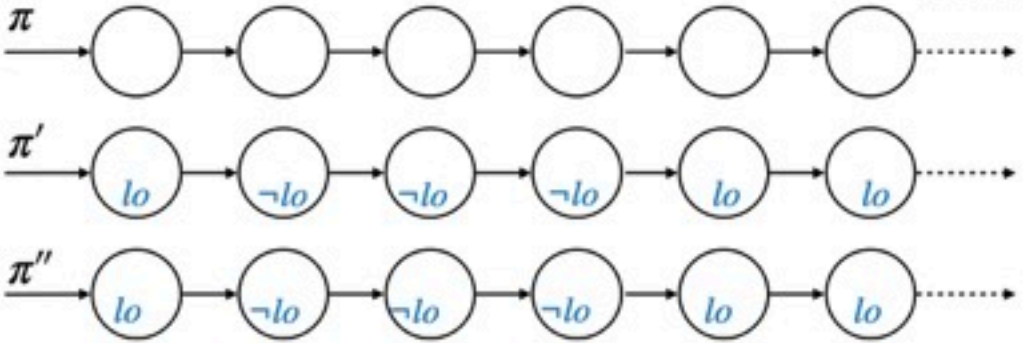

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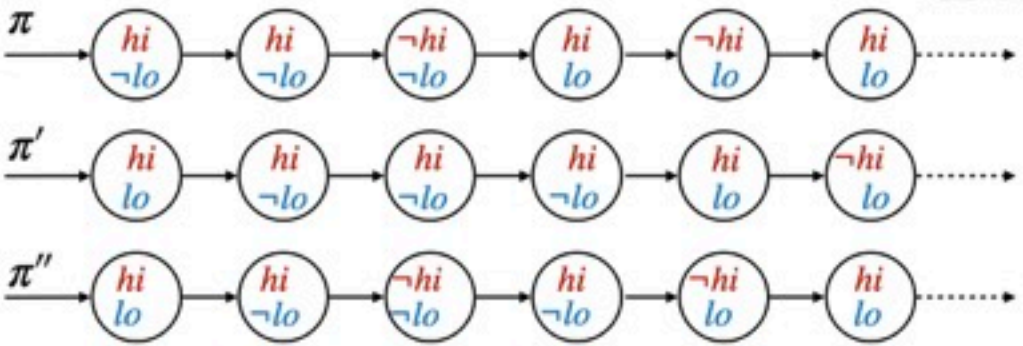
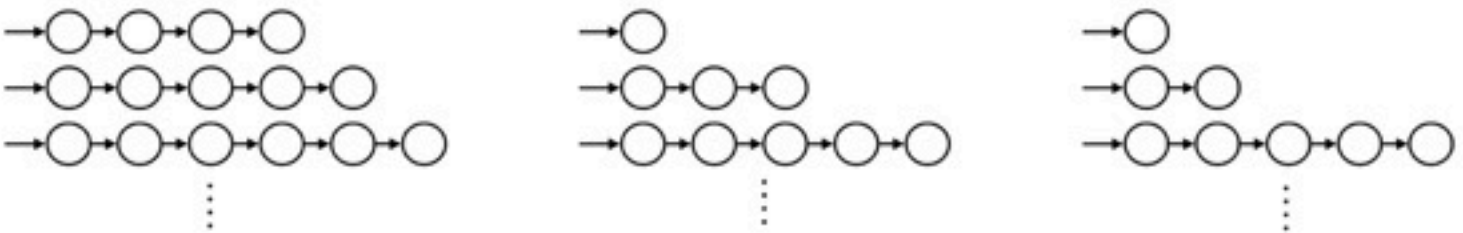


Model-Checking $\forall\exists$HyperLTL

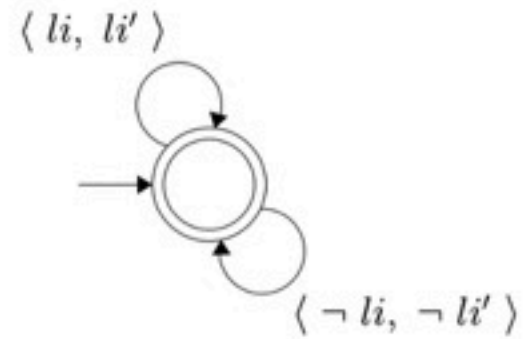
All infinite languages



for every trace
exists a longer trace
&
exists at least one trace

Model-Checking $\forall\exists$HyperLTL

$$\forall\pi\exists\pi' . \square(low - in_{\pi} \leftrightarrow low - in_{\pi'})$$



2 quantifiers, alphabet is $\Sigma \times \Sigma$

Construct automaton for the inner formula

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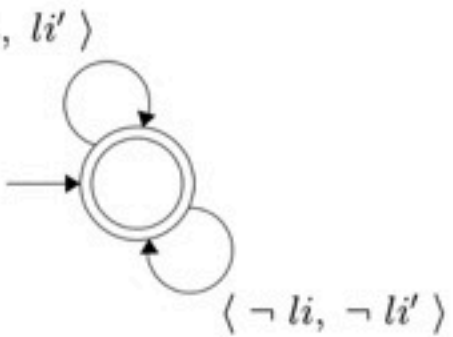
Model-Checking $\forall\exists$HyperLTL

$$\forall\pi\exists\pi' . \square(low - in_{\pi} \leftrightarrow low - in_{\pi'})$$

We cannot directly use negation since we have two types of quantifiers

Integrate the quantifier into the construction



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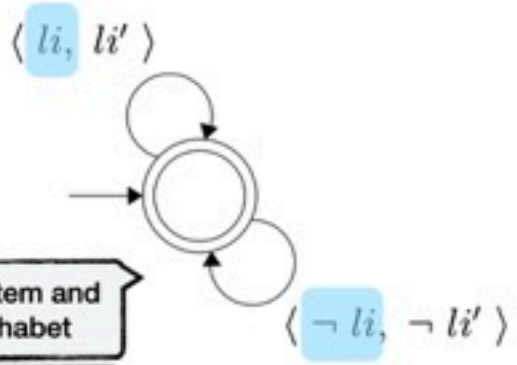

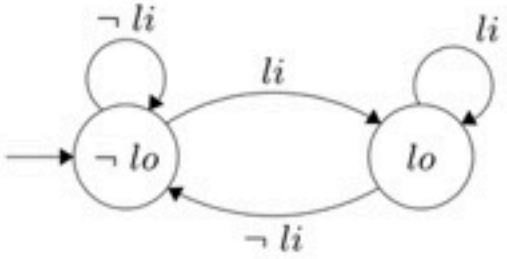

Intersect with the system and then reduce the alphabet

Integrate the quantifier into the construction

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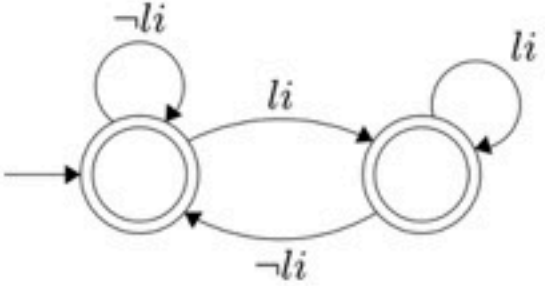

All traces for which there exists a trace such that...

Intersect with the system and then reduce the alphabet

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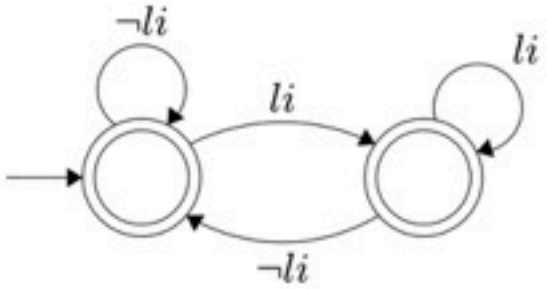

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∀ quantifier — looking for a violation

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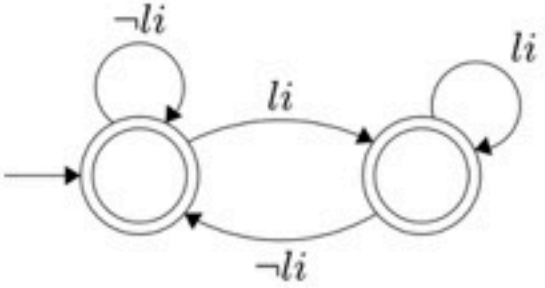

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Complement the automaton

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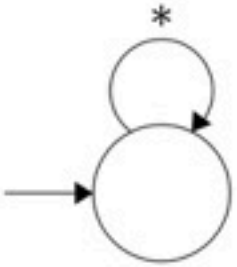

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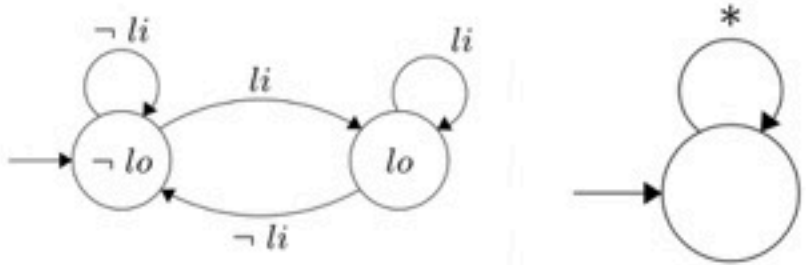

Intersect with the system to find a violation trace

All traces for which there exists a trace such that...

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Complement the automaton

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Model-Checking $\forall\exists$HyperLTL

$$\forall \pi_1 \exists \pi_2 . \psi$$

1. A_ψ — an automaton for the inner formula ψ, over 2-tuples of letters
2. M_k — a k self-composition of the system
3. Intersect $A_\psi \cap M_k$
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6. Check A' for emptiness

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Each alternation requiers complementation

Each alternation adds one exponent

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Model-Checking HyperLTL

$$\forall^* \exists^* \forall^* \exists^* . \psi$$

Each alternation requiers complementation

Each alternation adds one exponent

HyperLTL model-checking is tower-exponential

Inner formula into a non-deterministic Büchi automaton:
Exponential in the size of the inner formula

Every alternation:
Exponential in the size of the previous automaton

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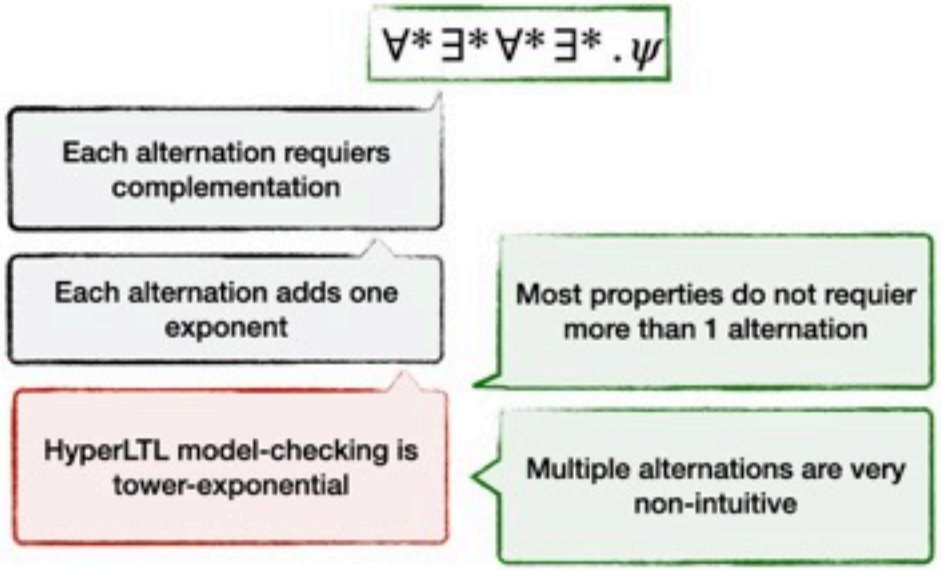
Model-Checking HyperLTL



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Demonstration

Hyperproperties are Important But Hard!

Outline

- Trace properties & Hyperproperties
- HyperLTL - a logic for temporal hyperproperties
- Some interesting properties in HyperLTL
- HyperLTL model-checking
  - $\exists^*$-HyperLTL
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Satisfiability of HyperLTL

Satisfiability of LTL

Given an LTL formula ψ
is there a trace that satisfies ψ?

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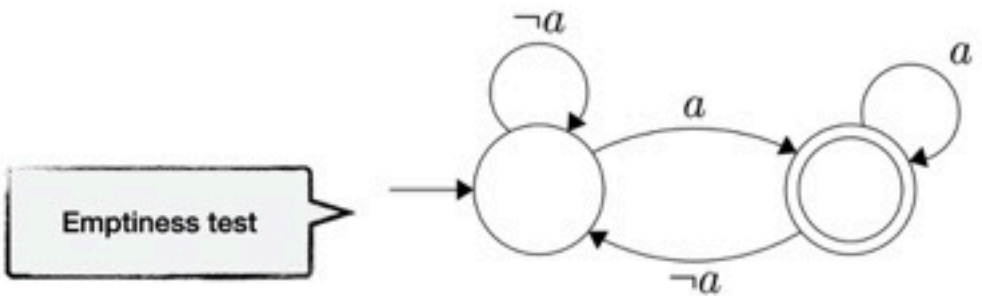
$\square\Diamond a$



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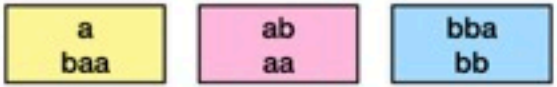

Deciding Hyperproperties. Finkbeiner and Hahn, CONCUR 2016

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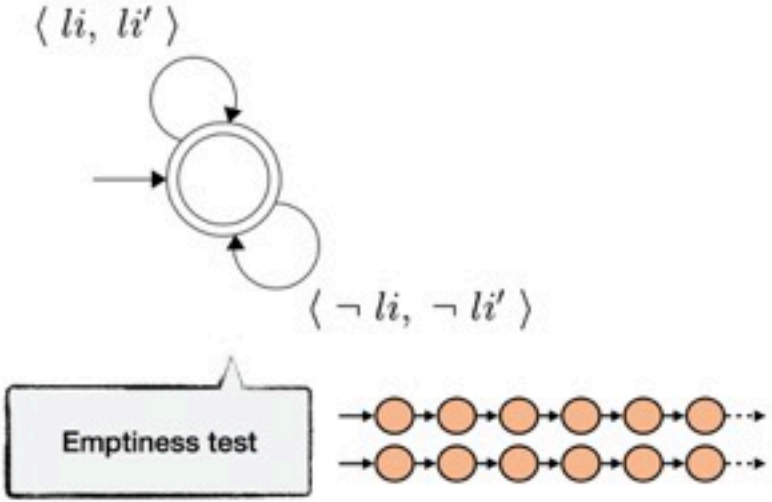

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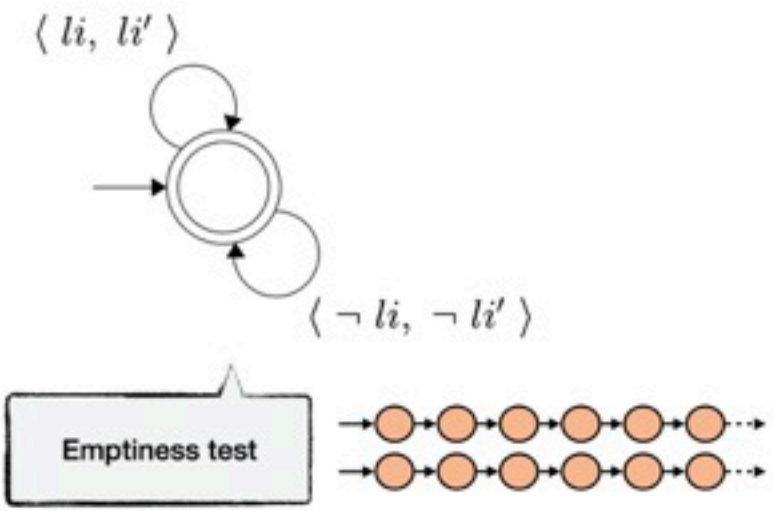

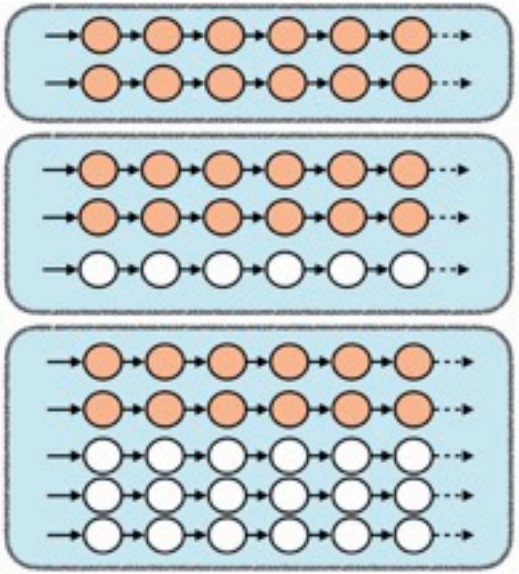

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If this holds, then it's true also for $\pi = \pi'$

It is enough to consider singelton models

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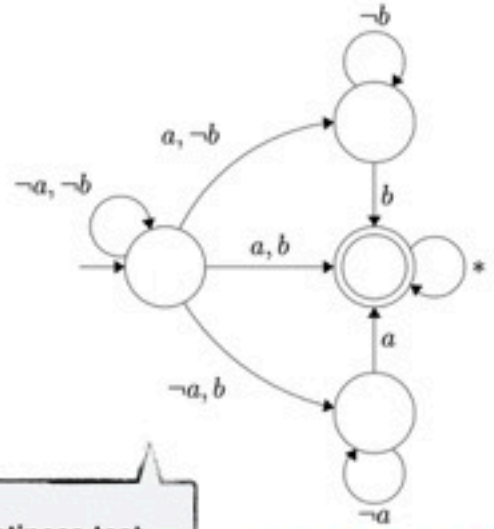
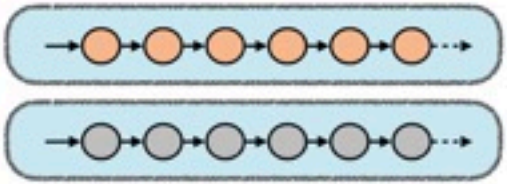
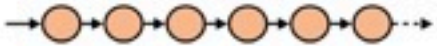




Emptiness test



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Satisfiability of $\exists^* \forall^*$ HyperLTL

$$\exists \pi \exists \pi' \forall \pi'' . \Diamond a_\pi \wedge \Diamond b_{\pi'} \wedge \Diamond c_{\pi''}$$

Consider all possible assignments for π''

$$\exists \pi \exists \pi' . (\Diamond a_\pi \wedge \Diamond b_{\pi'} \wedge \Diamond c_\pi)$$

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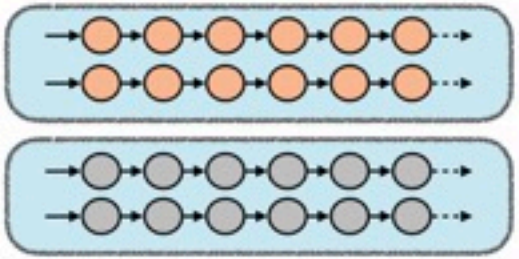

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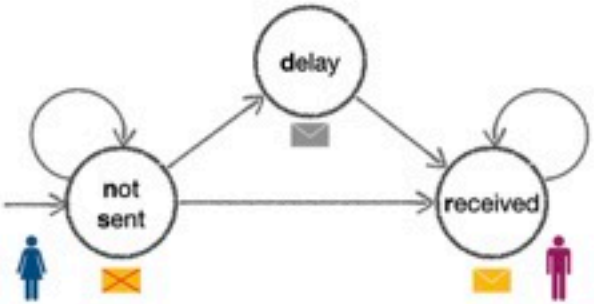
Knowledge and Common Knowledge in Multi-Agent Systems

Example: Knowledge



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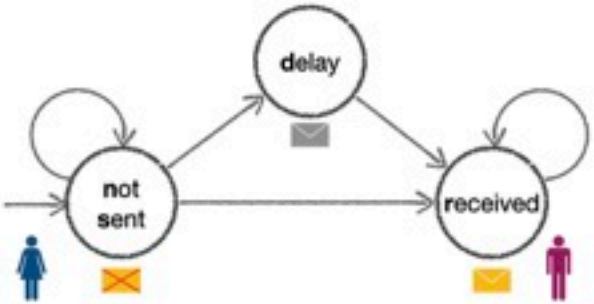

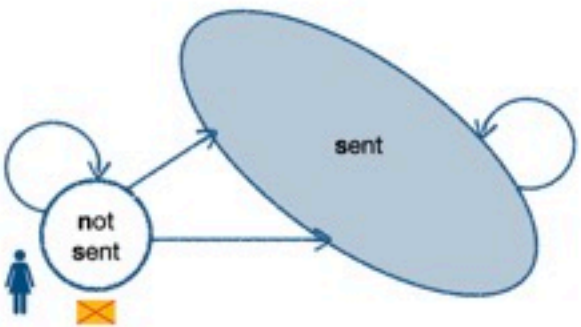

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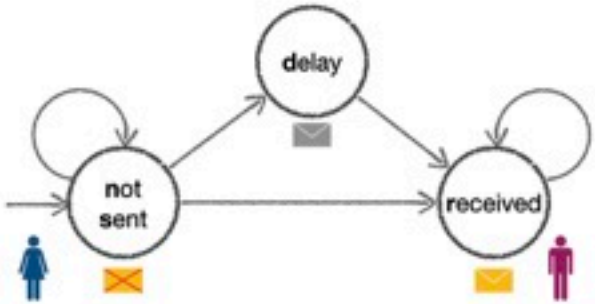
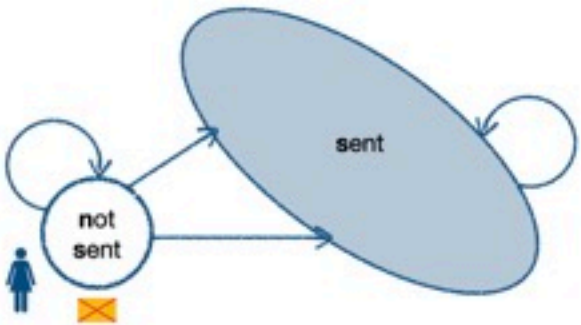
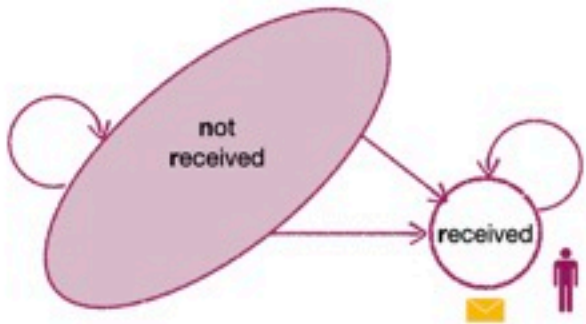
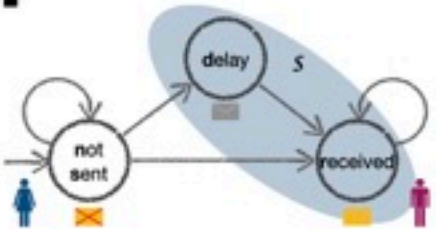
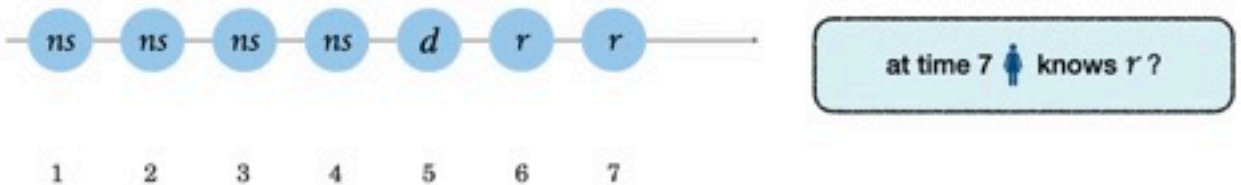




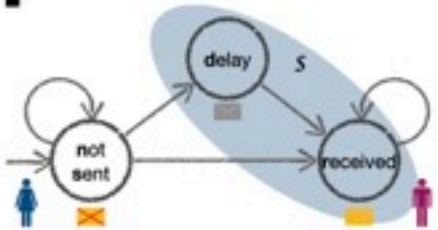


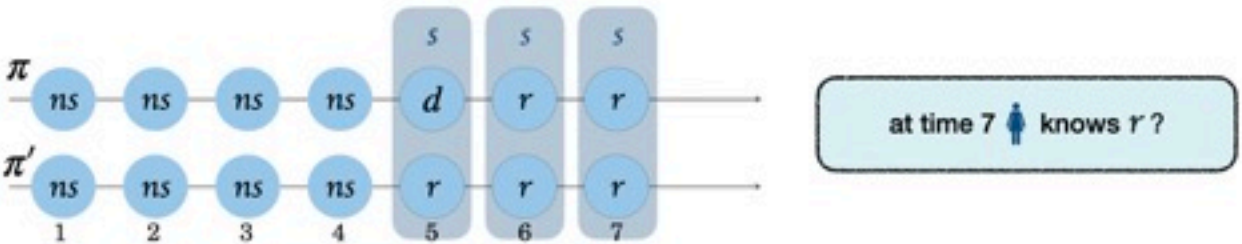
Example: Knowledge



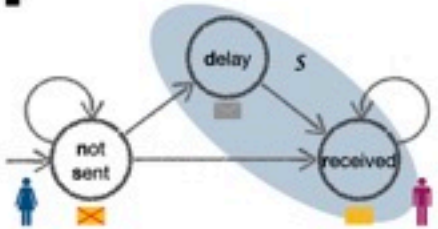


Example: Knowledge





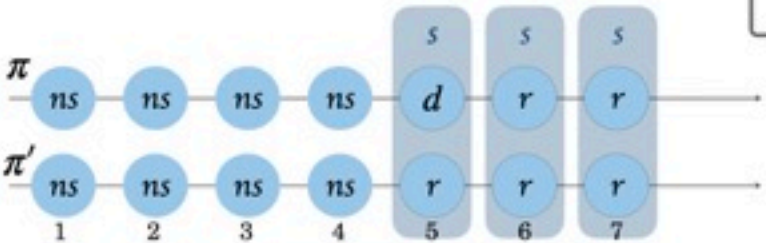
Example: Knowledge



Hyperproperty

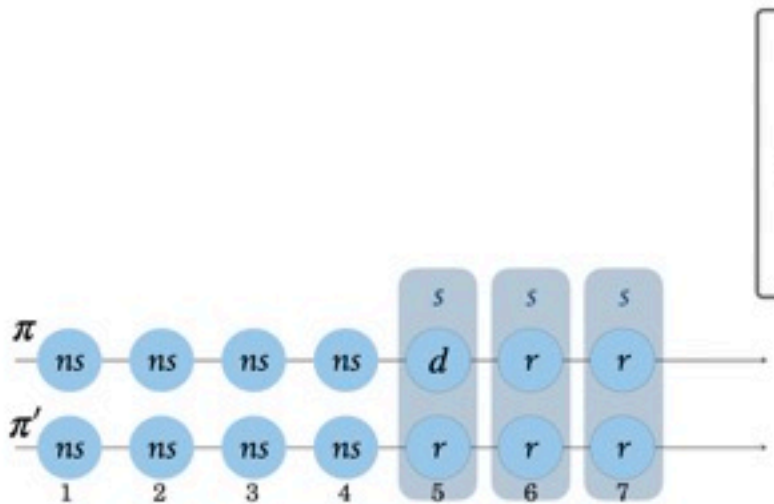
for every trace π'
that 🚹 cannot distinguish from π
until time 7
r holds at time 7

at time 7 🚹 knows r?



Knowledge in HyperLTL

$$\forall \pi \forall \pi' . (\pi \equiv_{\text{Alice}} \pi') \rightarrow \bigcirc^7 r_{\pi'}$$



Knowledge in HyperLTL

$$\forall \pi \forall \pi' . (\pi \equiv_{\text{🚹}} \pi') \rightarrow \bigcirc^7 r_{\pi'}$$

$$\bigwedge_{i=1..7} \bigcirc^i \left((ns_\pi \leftrightarrow ns_{\pi'}) \wedge (s_\pi \leftrightarrow s_{\pi'}) \right)$$

Hyperproperty

for every trace π'
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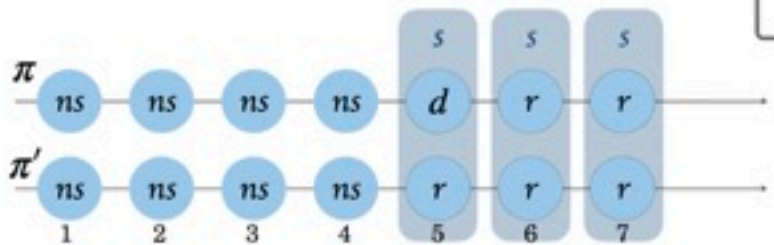
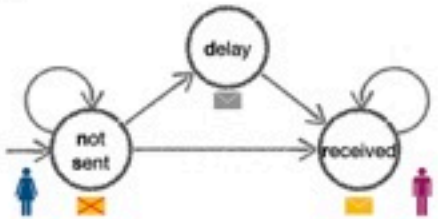
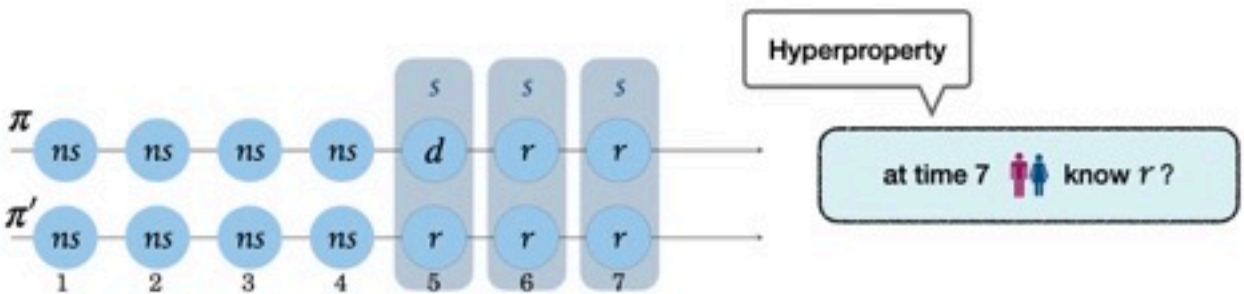
until time 7
r holds at time 7



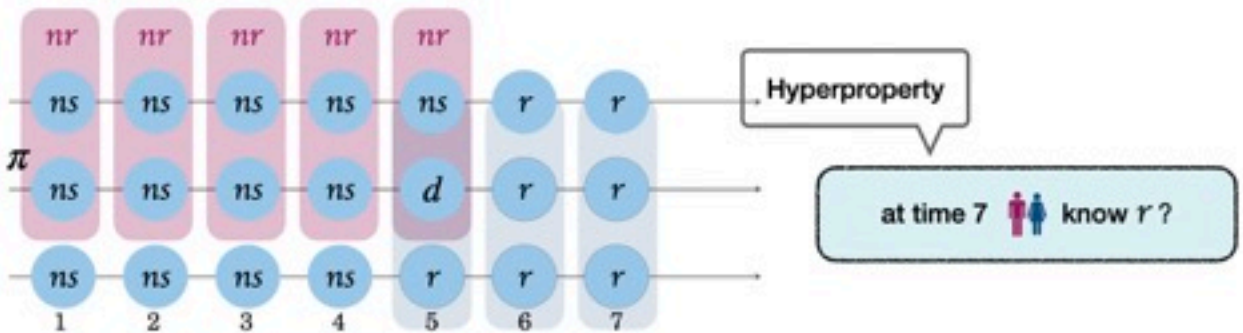
at time 7 🚹 knows r ?

Example: Knowledge

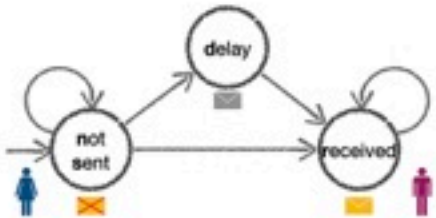




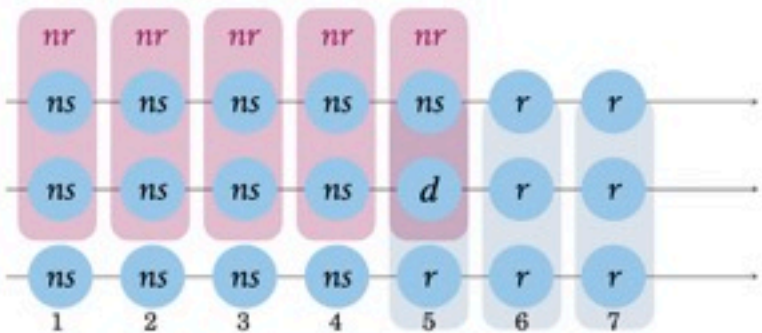
Example: Knowledge



Example: Common Knowledge



r is common knowledge at time 7 among  ?



Common Knowledge

φ common knowledge

$\leftrightarrow$

Reasoning About Knowledge. Fagin, Halpern, Moses, Vardi (MIT Press 1995)

Common Knowledge and Update in Finite Environments. van der Meyden (Inf. Comput. 1998)

Knowledge and common knowledge in a distributed environment. Halpern and Moses (J.ACM 1990)

Common Knowledge

φ common knowledge

first-order trace quantification is not enough

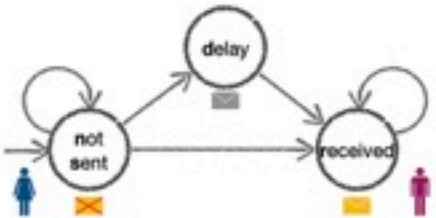
$\leftrightarrow$

know that know that know that

know that know that $\cdots$ φ

Unifying hyper and epistemic temporal logics. Bozzelli, Maubert, Pinchinat (FoSSaCS 2015)

Example: Common Knowledge



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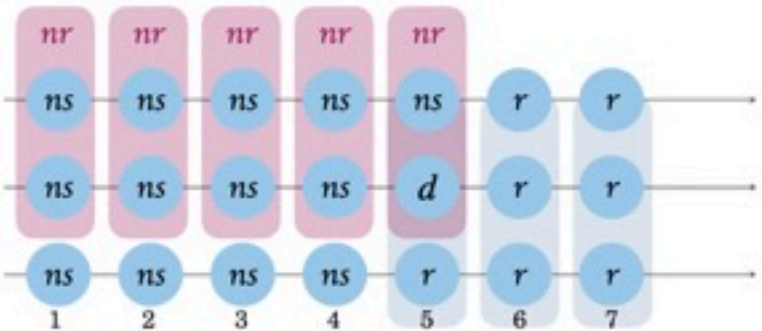

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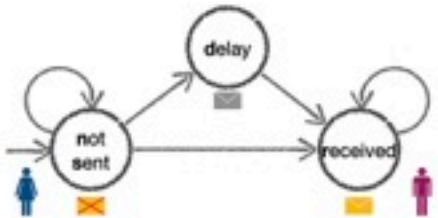

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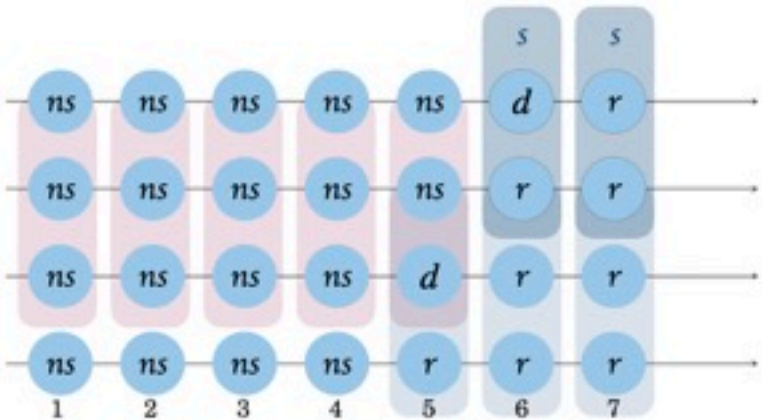

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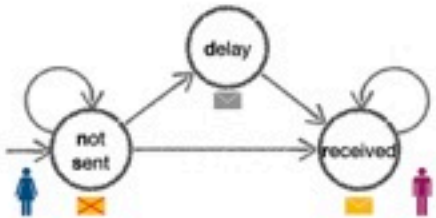

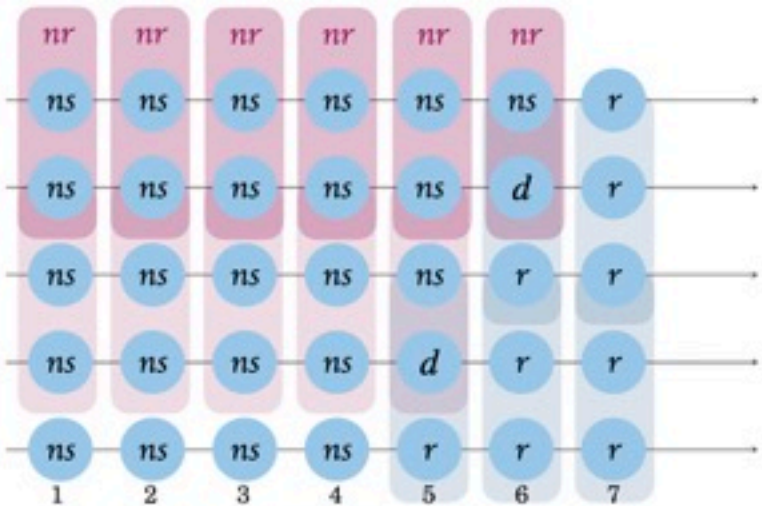

r is common knowledge at time 7 among 👫?

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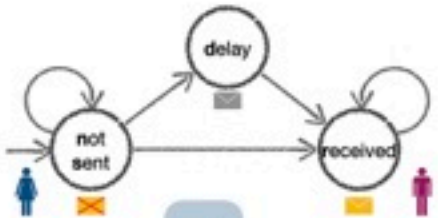

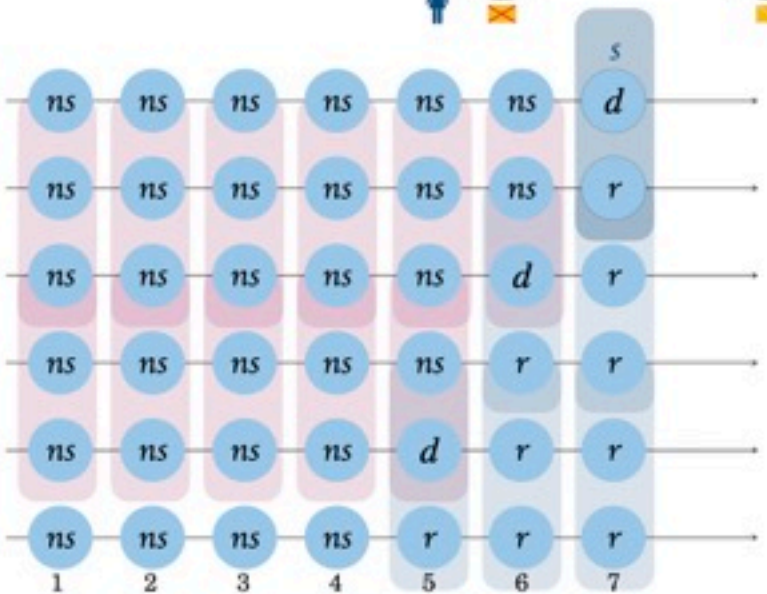

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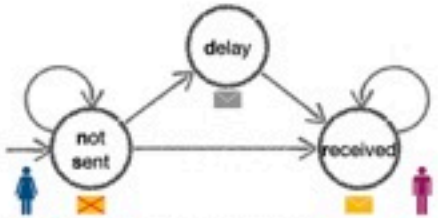

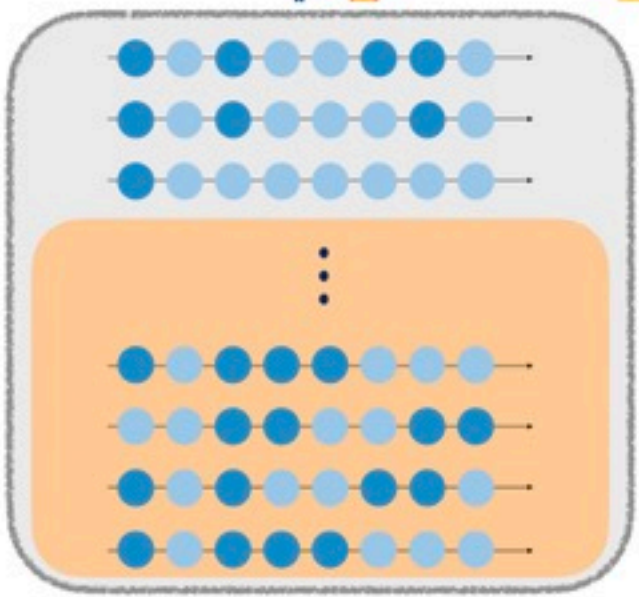

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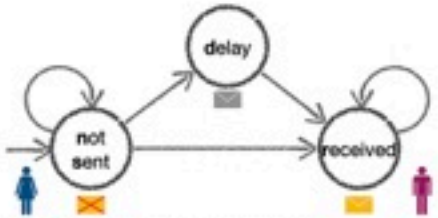
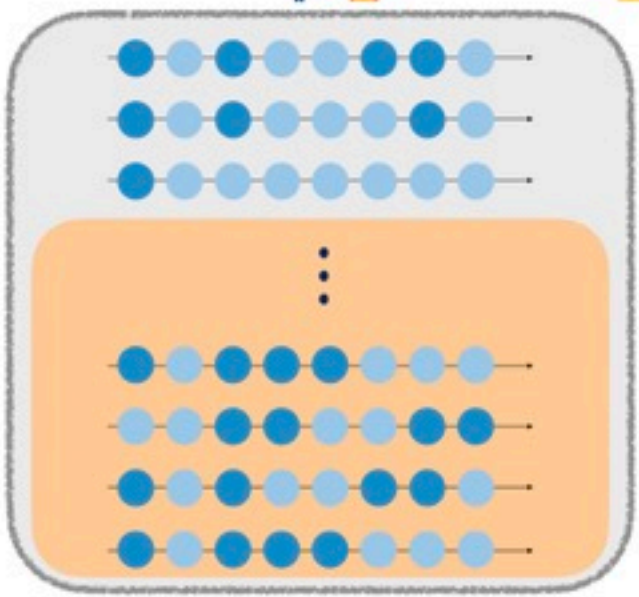


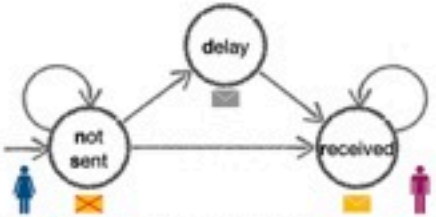


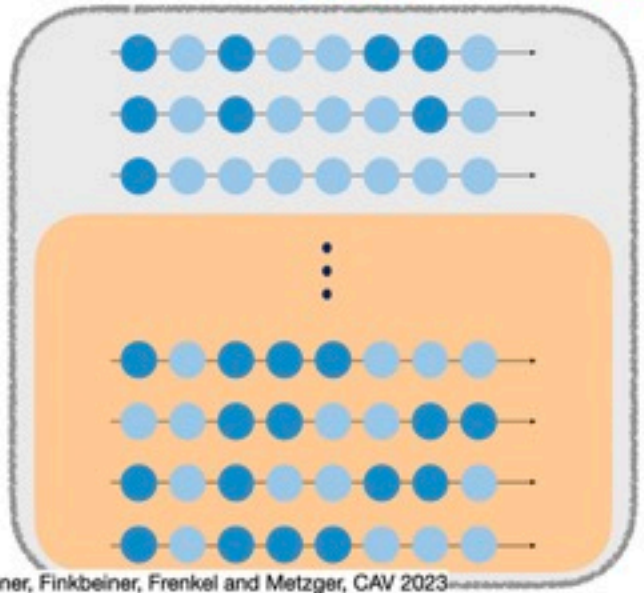
r is common knowledge at time 7 among  ?

1. Define the set X of traces of interest
   all indistinguishable traces

2. Verify the desired property on X
   r eventually holds

Example: Common Knowledge





Hyper^{2}LTL
Quantification over sets of traces

HyperLTL is not enough!

1. Define the set X of traces of interest
   all indistinguishable traces

2. Verify the desired property on X
   r eventually holds

Second-Order Hyperproperties. Beutner, Finkbeiner, Frenkel and Metzger, CAV 2023

Outline

- Trace properties & Hyperproperties
- HyperLTL - a logic for temporal hyperproperties
- Some interesting properties in HyperLTL
- HyperLTL model-checking
  - $\exists^*$-HyperLTL
  - $\forall^*$-HyperLTL
  - Quantifier alternation
- HyperLTL satisfiability
- Beyond HyperLTL and security

Summary

Hyperproperties are important!

Initially for information-flow, but not only

We need new formalisms to express hyperproperties

And verification algorithms…

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And verification algorithms…

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Captures a wide range of information-flow and security properties

Have a decidable (yet costly) model-checking algorithm

Summary

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And verification algorithms…

HyperLTL is a logic to express temporal hyperproperties

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Have a decidable (yet costly) model-checking algorithm

Can be used in practice when there are not too many quantifier alternations