Corporate Control Activism*

Adrian A. Corum Wharton

Doron Levit† Wharton

First draft: March 15, 2015

This draft: October 14, 2015

Abstract

This paper studies the role of activist investors in the market for corporate control. We show that activists have higher credibility than bidders when campaigning against entrenched incumbents, and hence, are more effective in relaxing their resistance to takeovers. This result holds although bidders and activists can use similar techniques to challenge the resistance of corporate boards (i.e., proxy fights) and have similar governance expertise. Since activists have a relative advantage in "putting companies into play", there is strategic complementarity between the search of activists for firms that are likely to receive a takeover bid and the search of bidders for targets with which they can create synergies and that are available for sale. The analysis sheds light on the interaction between M&A and shareholder activism and provides a framework to identify the treatment and the selection effects of shareholder activism.

KEYWORDS: Acquisition, Corporate Governance, Merger, Proxy Fight, Search, Shareholder Activism, Takeover.

JEL CLASSIFICATION: D74, D83, G23, G34

*The paper was previously titled as "The Role of Activist Investors in the Market for Corporate Control".

†The authors are from the University of Pennsylvania, Wharton School, Finance Department. For helpful comments, we thank Simon Gervais, Vincent Glode, Itay Goldstein, Andrey Malenko, Gregor Matvos, Christian Opp, Bilge Yılmaz, the participants at the 7th Summer Finance Conference at the IDC Herzliya, the Summer School on Financial Intermediation and Contracting at Washington University in St. Louis, and seminar participants at Wharton, Frankfurt School of Finance & Management, and Norwegian School of Economics. Emails: corum@wharton.upenn.edu and dlevit@wharton.upenn.edu.

"I'd like to thank these funds [Carl Icahn, Nelson Peltz, Jana Partners, Third Point] for teeing up deals because they're coming in there and shaking up the management and many times these companies are being driven into some form of auction." Thomas H. Lee, a private equity fund manager.[1]

1 Introduction

This paper studies the role of activist investors in the market for corporate control. The separation of ownership and control in public corporations creates agency conflicts between insiders and shareholders (Berle and Means (1932)). In order to protect their private benefits of control,[2] corporate boards can resist takeovers that would otherwise create shareholder value, for example, by issuing shareholder rights plans ("poison pills"). With a de facto veto power,[3] the resistance to takeovers can be overcome only if the majority of directors are voted out in a contested election ("proxy fight"). In fact, the power of shareholders to unseat directors is often used by the courts as the basis for allowing boards to block takeovers in the first place (Gilson (2001)).

Activist investors often demand from companies they invest in to sell all or part of their assets (Brav et al. (2008), Greenwood and Schor (2009), Becht et al. (2015)), and if needed, use proxy fights to force them to do so. For example, in 2014, the board of PetSmart agreed to be bought out for $8.7 billion after facing months-long pressure, which included the threat of a proxy fight, from one of its largest shareholders, the activist hedge fund Jana Partners.[4] In 2013, the private-equity firm KKR acquired Gardner Denver for $3.7 billion after ValueAct Capital accumulated a 5% stake in the company and agitated for its sale. Commenting on the deal, KKR's co-CEO, George Roberts, said: "We wouldn't have bought Gardner Denver had

[1] See The New York Times, "Will Credit Crisis End the Activists' Run?" 8/27/2007.

[2] Jenter and Lewellen (2015) provide evidence consistent with managers being reluctant to relinquish control due to career concerns. See also Walkling and Long (1984), Martin and McConnell (1991), Agrawal and Walkling (1994), Hartzell et al. (2004), and Wulf and Singh (2011)), who show that target CEOs typically suffer from poor career prospects following takeovers.

[3] Under most jurisdictions, including Delaware, merger proposals can be brought to a vote for a shareholder approval only by the board of directors. Alternatively, tender offers do not require a vote, but they are vulnerable to poison pills, which can be adopted on short notice and make a takeover virtually impossible.

[4] See The New York Times, "Under Pressure From Jana, PetSmart Says It Will Explore Potential Sale", 8/19/2014; The New York Times, "Elliott and Jana, Activist Investors, Are Behind 2 Big Buyouts", 12/15/2014; and The Deal Pipeline, "Jana Unveils Potential Board Slate for PetSmart", 11/21/2014.

not an activist shown up."[5] Consistent with these anecdotes, Greenwood and Schor (2009) find that a takeover is twice as likely if an activist hedge fund is a shareholder of the target.

Altogether, the evidence suggests that shareholder activism is an integral part of the M&A market. *However, since bidders and activist investors can use similar techniques to challenge corporate boards (i.e., proxy fights), the incremental role played by activists in this market is in fact unclear.* What is the relative advantage of activists, if any? What are the implications of activist interventions for the M&A market? Do activists complement the effort of bidders to acquire companies, or do they compete away their rents from takeovers? Moreover, establishing a theoretical foundation for the role of activists, which is the main objective of this paper, can help to distinguish between instances where activists are just selecting companies that are likely to receive a takeover bid and instances where their interventions affect the takeover process.

We study these questions by analyzing a dynamic bargaining model in which the identity of the target board is endogenized by an interim proxy fight stage. Initially, a bidder is negotiating a deal with the incumbent board of the target. Circumventing the board by making a tender offer to the target shareholders is not feasible. The board can use its veto power and reject takeover offers in order to protect its private benefits of control, even if the offer increases shareholder value. However, if the negotiations fail, a proxy fight to replace the board can be initiated either by the bidder or by an activist investor, who is a shareholder of the target. Winning a proxy fight is not trivial, as the challenger must convince the majority of target shareholders that replacing the incumbents with his nominees is in their best interests. If the proxy fight succeeds, the winning team obtains control of the target board, and a second round of negotiations between the bidder and the newly elected directors takes place. If no proxy fight is launched or if the proxy fight fails, the incumbent board retains control of the target and can use his authority to block the takeover.

Our first result shows that although both bidders and activists can launch a proxy fight, only activists can effectively use this mechanism to challenge the resistance of incumbent directors and facilitate the takeover. This result is consistent with the evidence that, unlike activists, hostile bidders rarely launch proxy fights.[6] The activists' relative advantage stems

[5] See The Wall Street Journal, "Activist Investors Gain in M&A Push," 1/5/2013.

[6] While proxy fights can be effective as threats, as they are in our model, their observed frequency can suggest on their empirical relevance. Fos (2015) documents 632 proxy fights between 2003 and 2012, out of which only 5% were sponsored by corporations (i.e., potential bidders), 70% by activist hedge funds, and the rest by other shareholders of the firm. Moreover, he finds that while the frequency of hostile tender offers decreased over the sample period, the frequency of proxy fights increased. See also Mulherin et al. (1998).

from their higher credibility when campaigning against the incumbents. To understand this observation, which is a novel aspect of our analysis, note that a proxy fight is not a referendum on the terms of the takeover, but rather a vote on the composition of the board. Winning a proxy fight does not compel the newly elected directors to execute the takeover at the initial terms. Once the bidder's nominees are elected to the board, the bidder, who is the counterparty to the transaction, will be tempted to abuse his control of the target board, exploit its access to private information, divert resources, and low-ball the takeover premium. *This is the commitment problem in hostile takeovers.* Without a commitment to act in their best interests, target shareholders, who rationally anticipate this opportunistic behavior, are unlikely to elect the bidder's nominees to the board.[7] By contrast, the activist buys a stake in the target with the expectation that the firm will be acquired. Unlike the bidder, the activist is on the sell-side like other shareholders of the target and has incentives to negotiate the highest takeover premium possible. Therefore, shareholders trust the activist and elect her nominees to the board, even without a firm commitment to act in their best interests. With the support of target shareholders, the activist can disentrench the incumbent board and help the bidder to complete the acquisition at a fair price. This is the added value of activist investors to the market for corporate control.

The unique ability of activists to relax the opposition of incumbents to takeovers crucially depends on the belief of target shareholders that the activist is on their side of the negotiating table. This observation has two implications. First, collaborations between activist investors and bidders are likely to fail, as they raise concerns that the activist is in fact on the buy-side of the transaction. As an example, in Section 3.1 we discuss the failed acquisition attempt of Allergan by Valeant and Pershing Square in 2014. More generally, our analysis suggests that the resistance of incumbents to takeovers can be overcome only if the capacity to disentrench the target board is separated from the capacity to increase value through acquisitions. Second, in many cases, institutional investors who hold diversified portfolios, e.g., mutual funds, own stakes both in the target and the acquirer (Matvos and Ostrovsky (2008) and Harford et al. (2011)). Since these investors are both on the sell-side and the buy-side of the transaction, they have less credibility than an activist who is purely on the sell-side. Our analysis therefore suggests that these diversified investors are unlikely to be effective in exercising corporate control activism even if they own large stakes in the target and have the governance expertise

[7] See Section 3.1.2 for a discussion on the various mechanisms that can partially alleviate the bidder's commitment problem.

that is needed to run a proxy fight.

In order to study the implications of interventions by activist investors, we endogenize the bidder's arrival and the activist's decision to become a shareholder of the target by augmenting the model with an initial search stage. Bidders have to search in order to identify companies with which they can create synergies, while activists have to search for companies that are likely to be a takeover target. After the search stage, the activist decides in which firms to invest by trading with a market maker à la Kyle (1985), and the bidder decides with which firm to start takeover negotiations, as described above.

Our second result shows that there is strategic complementarity between the search of activist investors for firms that are likely to receive a takeover bid and the search of bidders for potential takeover targets with which they have synergies. Intuitively, since search is costly (e.g., hiring advisors to carry out due diligence), bidders will search for targets only if they believe that these companies are also available for sale. Since activist investors have an advantage in relaxing the opposition of incumbents to takeovers, bidders have stronger incentives to search for a target if the target is likely to have an activist investor as a shareholder. At the same time, the activist can profit from searching and buying shares of potential targets only if these companies eventually receive a takeover offer from the bidder. Therefore, the incentives of the activist to search also increase with the search of bidders for takeover targets.

Strategic complementarity has several implications. First, the aggregate volume of M&A is positively related to the intensity of shareholder activism, which can be measured by the volume of 13D filings. Second, activist investors not only facilitate takeovers once the offer is on the table, but they also increase the likelihood that companies become a takeover target in the first place. Therefore, activists affect corporate control outcomes even if ex-post their threat of running a proxy fight is not credible. Third, small regulatory changes, such as easing the access of shareholders to the ballot or modifying the rules that govern the filing of 13D schedules, have an amplified effect on the aggregate volume of M&A. Fourth, strategic complementarity gives rise to multiple equilibria, ranked by the aggregate volume of M&A. That is, the market for corporate control can experience episodes of high volume ("hot markets") and low volume ("cold markets"), without any apparent changes in the underlying fundamentals of the economy. In this respect, the extent of M&A activity is self-fulfilling and unpredictable. Finally, policies and regulations that exclusively undermine shareholder activism, such as the legalization of two-tier "anti-activism" poison pills, will adversely affect M&A even if "standard

pills" that prevent unwanted takeovers are already prevalent.[8]

In our model, activists invest either because they believe the company is likely to become a takeover target ("selection effect") or because they can facilitate its takeover ("treatment effect"). We provide necessary and sufficient conditions under which the treatment effect exists in equilibrium. We show that the model's comparative statics is sensitive to the existence of the treatment effect, a feature that can be used to create identification strategies for empirical research.[9] For example, if only the selection effect is in play, the volume of M&A decreases with the severity of the agency problems in target firms. This is intuitive, as with more private benefits of control the incumbents are more likely to resist takeover bids. However, this relationship can reverse when the treatment effect is in play. In this case, more resistance of incumbents to takeovers can result in a *higher* volume of M&A. Intuitively, the resistance to takeovers provides activist investors with more opportunities to profit from their ability to put firms into play. Due to the strategic complementarity, bidders will search more intensively and the aggregate volume of M&A will increase. Therefore, the treatment effect can be identified by a positive relationship between the severity of agency problems in the cross section of target firms and the volume of M&A.

We consider several extensions of the baseline model. We identify alternative channels through which activist investors complement the effort of bidders to acquire companies. We show that even if bidders can overcome the aforementioned commitment problem, activists are likely to have stronger incentives to run proxy fights (because of their higher governance expertise, their shorter investment horizon, and the expectation that most of the surplus from the takeover is extracted by the target), and therefore, are more effective in facilitating takeovers. Moreover, activists can help bidders to win proxy fights by exercising their own voting rights and lobbying other shareholders. Alternatively, activists can solicit takeover offers from bidders either by informing them on the potential to acquire the target, or by reassuring them that they will face a weaker opposition to the takeover, if the offer is fair.

Finally, our analysis identifies two instances in which activists compete away the rents of bidders from takeovers. First, in management buyouts the incumbents may be too motivated to sell the firm, even if the deal compromises shareholder value. We show that activist investors

[8] In 2014, the Delaware court allowed Sotheby's to keep a unique two-tier poison pill that was purposely meant to block the activist hedge fund Third Point from increasing its ownership in Sotheby's above 10%. See THIRD POINT LLC v. Ruprecht, Del: Court of Chancery 2014.

[9] Apart from providing comparative statics on the probability of a takeover, we also analyze the average abnormal returns around the announcements of acquisition agreements and 13D filings by activists.

will challenge the deal by using their influence on target shareholders to either block the transaction or "force" the bidder to sweeten the bid. Second, activist investors may also have the expertise to propose and execute operational, financial, and governance related policies that increase the standalone value of the target. We show that by providing a viable alternative to the takeover, the activist can force the bidder to pay a higher takeover premium. While in both cases the bidder may have weaker incentives to search and acquire the target, the presence of the activist can still increase the expected value of target shareholders.

The paper is organized as follows. The rest of this section highlights the contribution to the literature. Section 2 presents the setup of the baseline model, and Section 3 provides the core analysis. Section 4 offers several extensions to the baseline model. Section 5 concludes. Appendices A, B, and C give all proofs and results not in the main text.

Related Literature

Our paper connects the literature on blockholders and shareholder activism (for a survey, see Edmans (2014)) with the literature on takeovers (for a survey, see Becht et al. (2003)). Unlike models where the bidder is also a blockholder of the target prior to the takeover (e.g., Shleifer and Vishny (1986), Hirshleifer and Titman (1990), Kyle and Vila (1991), Burkart (1995), Maug (1998), Singh (1998), and Bulow et al. (1999)), here the activist, who is a shareholder of the target, can pressure the incumbent board to accept a takeover offer, but she cannot or does not have incentives to acquire the target herself. In fact, our analysis, which identifies the commitment problem in hostile takeovers, emphasizes the benefit from separating the capacity to disentrench boards from the capacity to increase firm value through acquisitions.

Several papers have focused on the interaction between bidders and large shareholders of the target company. Cornelli and Li (2002) study a model in which arbitrageurs accumulate large stakes in the target and mitigate the free-rider problem of Grossman and Hart (1980) by tendering their shares to the bidder. Gomes (2012) studies a dynamic model of tender offers in which the arbitrageurs, by holding blocks of shares, force the bidder to make a high preemptive bid to counter a credible hold-out. Burkart et al. (2000) develop a model in which the bidder chooses between a privately negotiated block transfer with the target's leading minority shareholder and a public tender offer, and show that the mode of transaction matters. In a contemporaneous work by Burkart and Lee (2015), an activist shareholder of the target can relax the free-rider problem in tender offers by directly negotiating an acquisition agreement

with the bidder. Different from these studies, we abstract away from the free-rider problem in tender offers. Importantly, we allow the incumbent board to veto any offer made directly to shareholders, for example, by issuing a poison pill, and focus the analysis on the agency conflicts between the target board and its shareholders,[10] as well as on the search friction in the market for corporate control. We show that activist investors have an advantage relative to bidders in disentrenching corporate boards, a feature which gives rise to strategic complementarity between activist's and bidder's search efforts.

Various aspects of proxy fights within and outside the context of takeovers have been analyzed in the literature (e.g., Shleifer and Vishny (1986), Harris and Raviv (1988), Bhattacharya (1997), Maug (1999), Yılmaz (1999), Bebchuk and Hart (2001), and Gilson and Schwartz (2001)). In none of these papers, however, can an activist investor who is not the bidder launch a proxy fight to replace the incumbent directors of the target. Here, both the bidder and the activist can challenge the board. Our observation that activist investors use proxy fights more effectively than bidders to relax the opposition of incumbents to takeovers is a novel aspect of our analysis.

2 Setup

Consider an economy with a bidder, an activist investor, and $N \geq 2$ ex-ante identical public firms. The standalone value of each firm is $q > 0$, which is common knowledge. Initially, each firm is owned by passive shareholders (institutional or retail) and run by its board of directors. We normalize the number of shares of each firm to one. Each share carries one vote. According to the governance rules of each firm, a successful takeover requires at least half of its voting rights. All agents are risk-neutral.

The incumbent board of each firm has private benefits of control (e.g., excessive salaries, perquisites, investment in 'pet' projects, access to private information, pleasure of command, prestige, or publicity), which are lost if the firm is acquired or if shareholders elect a new board. We do not distinguish between the manager and other board members, and treat the board as a monolithic entity. Consistent with Jenter and Lewellen (2015), we assume that compensation contracts cannot fully align the incentives of the board. We denote the board's private benefits

[10] Models in which the target board can resist a takeover offer have also been studied by Bagnoli et al. (1989), Baron (1983), Berkovitch and Khanna (1990), Hirshleifer and Titman (1990), Harris and Raviv (1988), and Ofer and Thakor (1987).

per share (owned by the board) by $b > 0$, where b is common knowledge.[11] Thus, from the incumbent board's perspective, the firm's standalone value is $q + b$ per share.

The bidder can add value through acquisition to exactly one of the N firms.[12] We refer to this firm as the *target*. The identity of the target is unknown, and initially, each firm is equally likely to be the target. If the bidder acquires the target, the added value net of any transaction cost is $\Delta \in [0, \infty)$. The probability density function of Δ is given by f and its cumulative distribution function is given by F. Both are continuous and have full support. If the bidder is a strategic acquirer (e.g., a corporation in a related industry) then Δ is the net operational or financial synergy with the target, and if the bidder is a financial acquirer (e.g., a private equity firm) then Δ is the net operational improvement that arises from a going private transaction or the net synergy with one of its portfolio companies. Δ can also include the bidder's private benefits from acquiring the target. While the acquisition of the target creates value, the acquisition of a non-target firm does not create value, and possibly wastes corporate resources such as management's attention and advisors fees. Moreover, the integration of companies often distracts employees, increases uncertainty, and requires additional compliance with regulation, all of which can be detrimental to firm value. We assume that the unconditional expected value creation from the acquisition of any of the N firms is negative. This assumption guarantees that the bidder will not approach any firm with a takeover offer without first verifying it is the target.

Unlike the bidder, the activist cannot add value through acquisition to any firm, including the target. Therefore, the activist has no incentives to make a takeover bid. Alternatively, the activist does not have enough capital to make a takeover bid, an assumption which is consistent with Brav et al. (2008) who show that hedge fund activists seldom seek control themselves. Consistent with Greenwood and Schor (2009) and Becht et al. (2015), who show that the positive abnormal returns around 13D filings by activist investors stem mostly from events in which the target is eventually acquired, we also assume that the activist cannot affect any of the firms' standalone value. We relax this assumption in Section 4.2.2, where among other things, we show that activists are more resilient than bidders to the aforementioned commitment problem even if activists were allowed to make takeover bids. Moreover, consistent with the common critique that activist investors have a short-term investment horizon (for example,

[11] The main results do not change qualitatively if b is unknown at the search stage, but is revealed once the negotiations start.

[12] The focus of the analysis is on the sale of the entire firm, but it can be applied to divestitures or spinoffs.

because of their desire to establish reputation, higher alternative cost of capital, or the need to meet interim fund out-flows), we assume that the activist discounts the standalone value of each firm by $1-\gamma \in [0,1]$. Since the proceeds from a takeover are received by shareholders before the standalone value of the firm is realized, this assumption implies that relative to other investors, the activist is biased toward selling the firm.

Figure 1 depicts the timeline of the model, which is described in detail below. The first phase endogenizes the arrival of the bidder and the activist's decision to become a shareholder of the target. The second phase includes the dynamic bargaining between the bidder and the target board, where the identity of the target board is endogenized by an interim proxy fight stage.

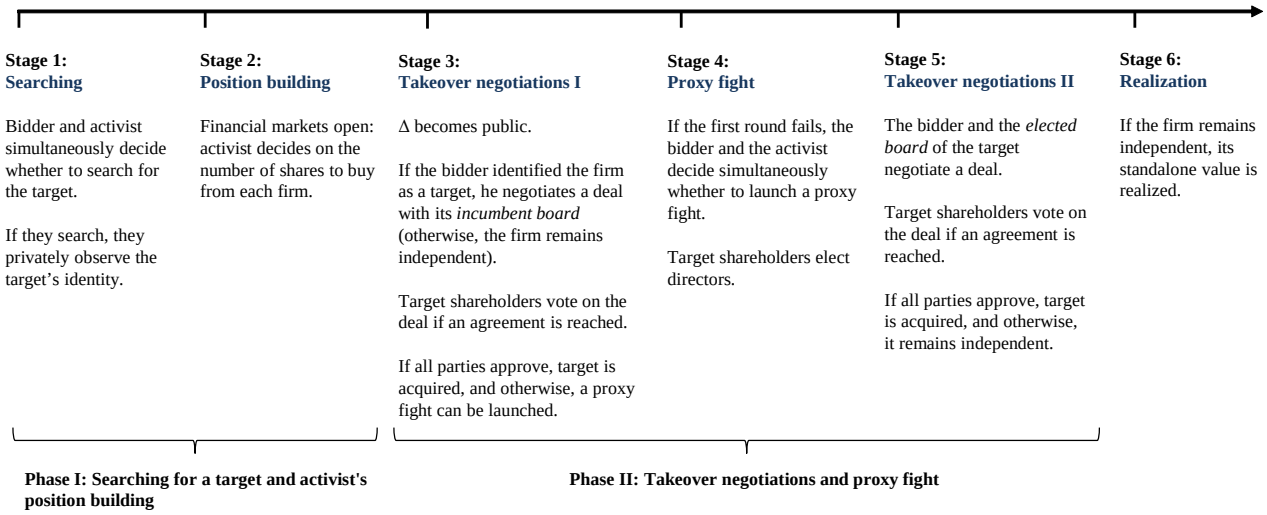


Figure 1 - Timeline

I. Searching for a target and activist's position building:

At the outset, the bidder and the activist, who do not own shares in any firm, have to search in order to learn which of the N firms is the target. They are also uninformed about Δ. Search is costly. We denote by c_B and c_A the search cost of the bidder and the activist, respectively. We assume that c_B and c_A are drawn from continuous cumulative distributions G and H, respectively. Both distributions have full support on $[0,\infty)$, and c_A and c_B are independent of each other and all other random variables. The bidder and the activist privately observe their search cost before they decide whether to search. The search decisions are made simultaneously, and they are also the bidder's and the activist's private information. If the bidder and the

activist decide to search, they privately and perfectly learn the identity of the target.[13]

After deciding whether to search, the activist decides on how many shares to buy in each firm. Short sales are not allowed. The activist trades without knowing whether the bidder searched for the target and has intentions to make an offer. In Section 3.2.1, we discuss scenarios in which the activist trades after the negotiations between the bidder and the target become public. The activist trades with a risk-neutral and competitive market maker, who sets the prices equal to the expected value of the firm given the available information. Each firm i has a separate market maker, who privately observes the total order flows for the firm, denoted by $z_i \geq 0$. The order flows of firm i are either generated by the activist or by liquidity traders. The market maker cannot distinguish between the two. Liquidity trades are independent across firms. Specifically, with probability $\frac{1}{2}$ liquidity traders in firm i submit an order to buy $L > 0$ shares, and with probability $\frac{1}{2}$ they do not trade. We assume that purchasing L shares does not trigger a poison pill if such exists, and in particular, $L < 0.5$. We denote the share price of firm i by $p_i(z_i)$. After trading, the position of the activist in firm i, denoted by $\alpha_i \geq 0$, is observed by the market maker, the shareholders, and the board of firm i (e.g., by filing schedule 13D). By contrast, the bidder observes the position of the activist in firm i only if he searched and identified firm i as the target. This assumption implies that the bidder cannot free-ride on the activist's search effort, or alternatively, that the activist cannot directly solicit takeover bids. In Section 4.1.3, we relax this assumption and show that the main results continue to hold.

II. Takeover negotiations and proxy fight:

Since on average a takeover does not create value, if the bidder did not identify firm i as the target, he does not approach it with a takeover offer, and the firm remains independent. If the bidder identified firm i as the target, he starts negotiating an acquisition agreement directly with its incumbent board. This assumption reflects the ability of the board to block any attempt of the bidder to bypass the board and make a tender offer directly to target shareholders.[14] For simplicity and to focus the analysis on agency conflicts as the key friction, we abstract from information asymmetries about Δ. Specifically, we assume that Δ becomes public once the takeover negotiations start.[15]

[13] The assumptions on the search technology are made for simplicity. The main results continue to hold if instead the search cost is $c(\lambda)$, where $c', c'' > 0$ and λ is the probability the target is identified.

[14] We abstract from the free-rider problem in tender offers in the sense that the incumbent board cannot benefit from the implicit bargaining power that arises from the free-rider problem.

[15] The analysis does not change materially if information about Δ (or q) is incomplete but symmetric, if Δ is revealed to the activist and the target shareholders only at the end of the first round of negotiations, if the

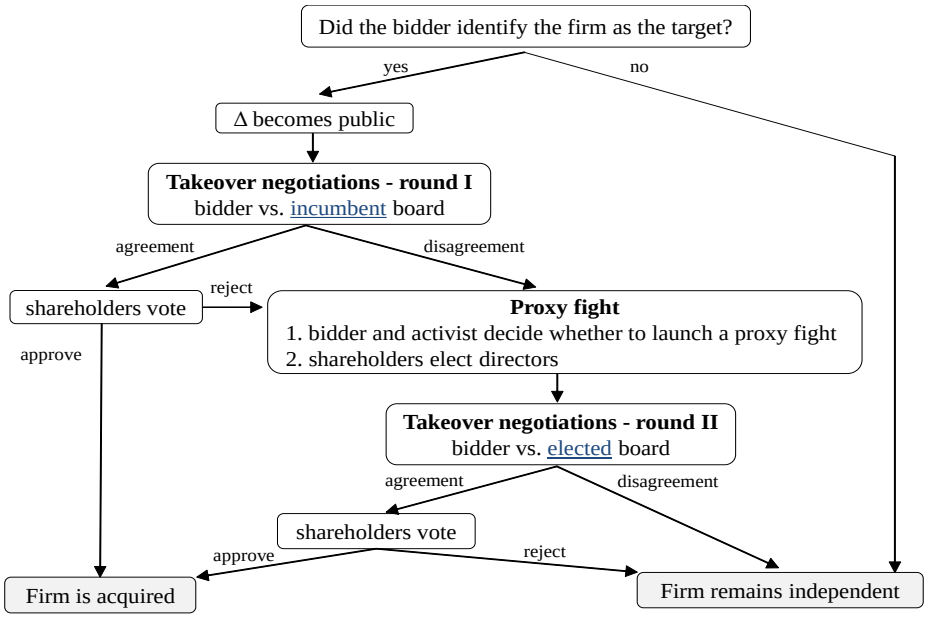


Figure 2 - Takeover negotiations and proxy fight phase

The parties negotiate a cash offer for 100% of the target shares. As depicted by Figure 2, there are two rounds of negotiations, indexed by $j \in \{I, II\}$, which are separated by a proxy fight stage. In each round, the proposer is decided randomly and independently from the other round. With probability $s \in (0,1)$ the proposer is the target board, and with probability $1-s$ the proposer is the bidder. The proposer makes a take-it-or-leave-it offer to the other party. Parameter s can be interpreted as the bargaining power of the target firm.[16] We denote the price per share paid by the bidder under an acquisition agreement that is reached in round j by π_j. If an agreement is reached, then it must be brought to a vote of the target shareholders and receive approval by a majority of them. We assume that target shareholders believe that their individual decisions cannot change the outcome of the vote, and at the voting stage they play undominated strategies. If the agreement is approved by shareholders, each shareholder, whether or not he voted for the acquisition, receives π_I for each share he owns, and the bidder gets $q + \Delta - \pi_I$.

bidder and the activist learn about Δ as they search, or if Δ is a commonly known from the outset. Generally, information asymmetries about Δ (or q) at the negotiations phase create adverse selection that could undermine both the bidder's and the activist's credibility when campaigning against the incumbents. Yet, since activists are on the sale-side while bidders are on the buy-side, activists are likely to have higher credibility even with information asymmetries, and hence, a relative advantage in relaxing the resistance of incumbent to takeovers.

[16] The bargaining protocol can be microfounded using Rubinstein's (1982) model of alternating offers.

If no agreement is reached at the first round, or if shareholders vote down a proposed agreement, the bidder and the activist decide simultaneously whether to run a proxy fight to replace the incumbent board.[17] The ability (or incentives) to run a proxy fight is a key feature that distinguishes the activist from other passive investors. If a proxy fight is initiated, the challenger incurs a non-reimbursable private cost $\kappa > 0$, which captures administrative costs as well as the effort, time, and money that are needed in order to recruit nominees, coordinate with other shareholders, and campaign against the incumbent. For example, κ decreases with the fraction of the firm that is held by institutional investors, or the governance expertise of the challenger. Target shareholders then decide whether to vote for the incumbent board or for one of the rival teams. Shareholders play undominated strategies when they elect directors, and the team that receives the largest number of votes is elected and takes control of the target board.

Winning control of the target board has two implications for the rival team. First, it gives the rival the right to negotiate on behalf of the target shareholders an acquisition agreement with the bidder in the second round. That is, the newly elected directors can redeem the poison pill, if such exists, and resume negotiations.[18] Second, the rival takes control of the operations of the target, and among other things, it can divert corporate resources as private benefits if the firm remains independent, for example, by exploiting the privileged access as a board member to the target's proprietary information or through self-dealing transactions.[19] We assume that the amount that can be diverted is limited and arbitrarily small. This assumption guarantees that if shareholders are indifferent between electing the rival (the bidder or the activist) and retaining the incumbent, they will choose the latter.[20] Importantly, both the bidder and the activist cannot commit to act in the best interests of target shareholders once they obtain control of the board. Under this assumption, the newly elected directors maximize the value of the party with which they are affiliated, even if it conflicts with maximizing target shareholder value. We discuss this assumption in Section 3.1.2 and relax it in Section 4.1.1.

[17] We implicitly assume that the majority of directors stand for reelection. In 2013, only 11% of the S&P 500 companies had a classified board, down from 57% in 2003 (see sharkrepellent.net: "Governance Activists Set Their Sights on Netflix's Annual Meeting" and "2003 Year End Review".) Taking full control of a staggered board requires winning two director elections, which can be prohibitively costly (e.g., Bebchuk et al. (2002)).

[18] Provisions that make pills nonredeemable are illegal in most states, including New York and Delaware.

[19] See Atanasov et al. (2014) for a discussion on the various forms of tunneling, and Atanasov et al. (2010), Bates et al. (2006), and Gordon et al. (2004) for evidence on tunneling in the U.S.

[20] In Appendix C.2, we show that the main results continue to hold when the rival's ability to divert corporate resources is not trivial.

Once the proxy fight stage ends, a second round of negotiations between the bidder and the target board (which may now be populated with the newly elected directors) takes place. The second round has the same protocol as the first round, and it is followed by a shareholder vote if an agreement is reached. If no agreement is reached, or if shareholders reject the deal, the target remains independent. If the firm remains independent, its standalone value is realized.

3 Analysis

We consider the set of Perfect Bayesian Equilibria in pure strategies and solve the game backward.

3.1 Takeover negotiations and proxy fights

We start by characterizing the second round of negotiations.

Lemma 1 *In the second round of negotiations, the target is acquired by the bidder unless the incumbent board retains control and $\Delta < b$. The shareholder value conditional on Δ is*

$$\Pi_{SH}(\Delta) = \begin{cases} q + \mathbf{1}_{\{b \leq \Delta\}} \cdot [s\Delta + (1-s)b] & \text{if the incumbent board retains control,} \\ q + s\Delta & \text{if the activist controls the board,} \\ q & \text{if the bidder controls the board.} \end{cases} \quad (1)$$

Several observations follow from Lemma 1. First, if reelected, the incumbent board can block the deal and consume his private benefits of control. Therefore, he would accept a takeover offer if and only if the premium is higher than b. If $b \leq \Delta$, the bidder can afford to pay a takeover premium of b. In this case, the entrenchment of the incumbent benefits target shareholders (at least ex-post) since it forces the bidder to offer a higher takeover premium without endangering the deal. However, if $\Delta < b$, the bidder would rather walk away from the negotiations. In this case, the entrenchment of the incumbent board results with an inefficient outcome: a value-increasing takeover is rejected.

Second, in spite of the activist's bias toward selling the target (whenever $\gamma > 0$), if she is elected to the board, the activist would negotiate a "fair" deal in which the bidder pays an expected takeover premium of $s\Delta$. To see why, note that the bidder will not offer less than q for the target. Indeed, while the activist cannot credibly reject offers higher than $q - \gamma q$,

any acquisition agreement must also be approved by the shareholders, who would reject offers lower than the perceived standalone value of the target, q. On the other hand, since $\alpha_i \geq 0$, the activist has incentives to maximize the value of her holdings, and therefore, whenever the activist is the proposer she would ask for $q + \Delta$, the highest price the bidder would agree to pay for the target.

Third, if the bidder wins the proxy fight then he obtains the authority to negotiate on behalf of the target shareholders. That is, the bidder is sitting on both sides of the negotiating table. Since the bidder has inherent incentives to acquire the target for the lowest price possible, he will take advantage of his power to offer target shareholders the lowest price they would accept, which is q. *This is the bidder's commitment problem in hostile takeovers.*

Lemma 2 *Suppose the first round of negotiations fails. Then:*

(*i*) *The bidder never runs a proxy fight.*

(*ii*) *If the activist owns α shares of the target, the activist runs a proxy fight if and only if*

$$\rho(\alpha) \leq \Delta < b, \tag{2}$$

where

$$\rho(\alpha) \equiv \max\left\{0, \frac{\kappa/\alpha - \gamma q}{s}\right\}. \tag{3}$$

Whenever the activist runs a proxy fight, she wins.

Lemma 2 establishes our result that although both bidders and activists can launch a proxy fight, only activists can effectively use this mechanism to challenge the resistance of incumbent directors and facilitate the takeover. According to part (i), the bidder does not run a proxy fight to replace the target board in any equilibrium of the subgame. This result holds regardless of the gains from the takeover, Δ, the cost of running a proxy fight, κ, whether or not the activist is also running a proxy fight, and the size of the incumbent board's private benefits of control, b. The reason is the following. As Lemma 1 suggests, because of the bidder's commitment problem, target shareholders are always worse off if they elect the bidder. Indeed, once elected, the bidder will be tempted to divert corporate resources and offer shareholders the lowest price possible. With rational expectations, shareholders would not elect the bidder's nominees to the board. Since running a proxy fight is both costly and inefficacious, the bidder will not run

a proxy fight. Note that this result holds even if the bidder had a toehold in the target, as a toehold does not change the incentives of the bidder to low-ball the takeover premium.

Consider part (ii) of Lemma 2. According to Lemma 1, if the activist controls the target board, she is expected to negotiate a takeover premium of $s\Delta$ in the second round. By contrast, if the incumbent board retains control and $b \leq \Delta$, shareholder expect the negotiated takeover premium to be $s\Delta + (1-s)\, b$. Therefore, shareholders reelect the incumbent. However, if $b > \Delta$ then shareholders expect the incumbent to block the takeover if he is reelected, and therefore, they elect the activist if she decides to run a proxy fight. Notice that, unlike the bidder, the activist has incentives to obtain the highest takeover premium when negotiating on behalf of the target, and therefore, shareholders elect the activist even if similar to the bidder she is tempted to the divert corporate resources in the event that the target remains independent. As we discuss below, being on the sell-side gives the activist an advantage relative to the bidder when campaigning against the incumbent.

The activist does not necessarily start a proxy fight even if she expects to win one. If the activist does not challenge the incumbent, the target remains independent and the value of her stake remains $\alpha\,(q - \gamma q)$. However, if the activist runs a proxy fight, the value of her stake increases to $\alpha\,(q + s\Delta)$. The activist runs a proxy fight if the resulted increase in value is higher than the cost of running a proxy fight, which holds if and only if $\frac{\kappa/\alpha - \gamma q}{s} \leq \Delta$. As expected, the activist is more likely to run a proxy fight when the target's bargaining power, s, is strong, the number of shares owned by the activist, α, is large, the activist's bias toward the takeover, γ, is significant, and the cost of running a proxy fight, κ, is small. Condition (2) is the intersection of the activist's incentives to run a proxy fight and the shareholders' incentives to support her in the challenge.

The next result summarizes the takeover negotiations and proxy fight phase and shows that the expected shareholder value (weakly) increases with the number of shares the activist holds in the target.

Proposition 1 *Suppose the bidder identifies firm i as a target and the activist owns α shares of that firm. Then, the unconditional shareholder value of firm i is $q + v\,(\alpha)$, where $v\,(\cdot)$ is an increasing function given by*

$$v\,(\alpha) = \int_{b}^{\infty} \left[s\Delta + (1-s)b\right] dF\,(\Delta) + \int_{\min\{b,\rho(\alpha)\}}^{b} s\Delta dF\,(\Delta)\,. \tag{4}$$

Once the bidder identifies firm i as the target, there are three cases to consider. First, if $b \leq \Delta$ then whether or not the activist is a shareholder of the target, the incumbent board reaches an agreement in which the bidder pays on average $q + s\Delta + (1-s)b$ per share and takes over the target after the first round of negotiations. This explains the first term in (4). Second, if $\rho(\alpha) \leq \Delta < b$ and the activist is a shareholder of the target then all parties involved understand that if no agreement is reached in the first round, the activist will launch a proxy fight to replace the incumbent, win the support of shareholders, and then negotiate on behalf of the target an agreement in which the bidder pays on average $q + s\Delta$ per share. In this region, the activist's threat of running a proxy fight is credible. Therefore, any first round offer below $q + s\Delta$ will be rejected by shareholders, and any offer above $q + s\Delta$ will be rejected by the bidder. The incumbent board understands that the takeover is inevitable, and he will accept any offer higher than $q + s\Delta$ in order to avoid the adverse consequences of losing the proxy fight (e.g., embarrassment or the loss of reputation). In this case, the bidder pays $q + s\Delta$ and takes over the target after the first round of negotiations.[21] This explains the second term in (4). Last, in all other cases, the incumbent board's entrenchment is high ($\Delta < b$) but the threat of a proxy fight is not credible ($b \leq \rho(\alpha)$). Therefore, the incumbent retains control of the board, maintains his resistance, and successfully blocks the takeover.

3.1.1 Discussion - the commitment problem in hostile takeovers

The contrast between parts (i) and (ii) of Lemma 2 emphasizes that even though the bidder and the activist have the same cost of running a proxy fight and the same tendency to divert corporate resources once elected, only the activist can effectively use proxy fights to relax the opposition of the incumbent board to the takeover. The lack of trust of target shareholders in the bidder's motives stems from the bidder being the counter party of target shareholders to the transaction. The advantage of the activist in relaxing the opposition of incumbents to takeovers crucially depends on the belief of target shareholders that the activist is indeed on their side of the negotiating table.

The observation above has two broad implications. First, the resistance of incumbents to takeovers can be overcome only if the capacity to disentrench the board is separated from

[21] Proxy fights are always off the equilibrium path; they are effective as threats. Instead, if the bidder or the incumbent board are uncertain about the activist's intention to run a proxy fight (e.g., unobserved heterogeneity in γ or κ), then proxy fights could appear on the equilibrium path, without significantly changing the main results.

the capacity to increase value through acquisitions. Therefore, collaborations between activist investors and bidders are likely to fail. A case in point is the unsolicited bid of Valeant to Allergan in 2014. Valeant teamed up with the hedge fund activist Pershing Square, with the intention that Pershing Square would build a significant toehold in Allergan and then push for its sale to Valeant. The sophisticated maneuver failed. Our analysis suggests that by teaming up with Valeant, Pershing Square lost its unique ability to relax the opposition of Allergan's board to the takeover, since shareholders of Allergan can no longer trust Pershing Square to act in their best interests once elected to the board. Shareholders of Allergan were likely concerned that Pershing Square was trying to advance the goals of Valeant at their expense. Without the trust of shareholders of Allergan, Pershing Square was as ineffective as Valeant in relaxing the opposition of Allergan's board to the proposed takeover.[22]

Second, large shareholders of the target can play the role of an activist in the context of takeovers only if they are truly on the sell-side of the transactions. Matvos and Ostrovsky (2008) and Harford et al. (2011) find that in many cases large target shareholders also hold large positions in the acquiring firm. With ownership on both sides of the transaction, these institutional investors lack the credibility that pure sell-side investors would have. Since the ability to win a proxy fight crucially depends on the credibility of the challenger, these investors are likely to be ineffective in relaxing the opposition of the board to the takeover. Therefore, our analysis suggests that large institutional investors with diversified portfolios (e.g., Vanguard, Fidelity, State Street, and BlackRock) are unlikely to play an active role in takeovers (at least when the bidder is a public corporation). This result holds even if these investors own large stakes in the target and have sufficient governance expertise.

3.1.2 Discussion - overcoming the commitment problem in hostile takeovers

Our analysis builds on the assumption that the newly elected directors maximize the value of the party with which they are affiliated rather than the value of target shareholders. The aforementioned advantage of the activist from having higher credibility exists as long as the bidder cannot perfectly and at no cost commit to act in the best interests of target shareholders once elected to the board.

In practice, however, there are several mechanisms and institutions that can potentially alleviate the bidder's commitment problem, but none of them seems perfect or costless. As

[22] Allergan was eventually acquired by Actavis, however, from the perspective of Valeant, the takeover attempt failed. See "The Flaws in Valeant's Activist Deal Effort", New York Times, 11/18/2014.

was mentioned above, a toehold cannot be a panacea as the bidder's incentives to low-ball the takeover premium do not change even if he owns shares in the target prior to making a bid. Instead, the bidder might try to recruit independent nominees to represent him on the target board. These nominees, however, may not only charge higher compensation, but may also be vulnerable to side payments from the bidder. Indeed, if the bidder can offer compensation contracts (explicit or implicit) that are unobserved by target shareholders, he will be tempted to incentivize the nominees to maximize his value even it involves sacrificing target shareholder value.

By contrast, effective investor protection laws and strong legal environment can help shareholders enforce directors' fiduciary duties, but litigation and enforcement are costly, uncertain, and limited to verifiable outcomes. Alternatively, serial acquirers or private equity funds, who repeatedly interact in the market for corporate control, might be able to develop reputation for not expropriating target shareholders. However, reputation can be fragile, it depends on the presence of public histories of past outcomes, and sometimes it can create unintended distortions. Competition (if exists) is another mechanism that can limit the bidder's ability to expropriate target shareholders. In practice, it may be hard to successfully low-ball the takeover premium if a superior competing bid is outstanding (e.g., the Revlon Rule under the Delaware corporate law). Yet, by controlling the target board, the bidder can still exploit his access to the target's private information and divert resources, thereby deterring competition. Finally, in the U.S., the bidder can run a proxy fight and at the same time make a tender offer that remains pending until after the elections. However, the bidder can amend the terms of the tender offer without restriction, at least as long as any of the conditions to the tender offer remains unsatisfied.[23] Even if the bidder can commit not to revise the tender offer, by doing so, he is exposed to the free-rider problem of Grossman and Hart (1980).[24]

Either way, in Section 4.1.1 we analyze an extension of the model in which the bidder does not suffer from the aforementioned commitment problem and discuss alternative channels through the activists complement the effort of bidders to acquire firms.

[23] Since the tender offer is made prior to the proxy fight, it typically has a condition that the offer is valid only if the poison pill is redeemed. Therefore, the newly elected directors can always choose not to redeem the pill, thereby paving the way for the bidder to revise the offer.

[24] Bebchuk and Hart (2001) propose amending the existing rules governing mergers to allow acquirers to bring a merger proposal directly to a shareholder vote without the approval of the board of directors. Under these rules, the bidder can effectively commit to a certain acquisition price.

3.2 Searching for a target and activist's position building

Consider the activist's decision to buy shares in firm i and the equilibrium stock price. The informational advantage of the activist stems from knowing which firm is likely to receive a takeover offer. Therefore, in equilibrium, the activist does not buy shares of any firm unless she first identified it as the target. As the next result shows, if the activist identifies firm i as the target, she would buy exactly L shares in order to disguise her trade as a liquidity and uninformed demand.

Lemma 3 *Consider an equilibrium in which the bidder and the activist search for the target with probability $\lambda_B \in [0,1]$ and $\lambda_A \in [0,1]$, respectively. The activist buys shares of firm i if and only if she searched and identified it as a target, in which case, the activist buys L shares. The share price of firm i in equilibrium is given by,*[25]

$$p_i\left(z_i;\lambda_A,\lambda_B\right)=q+\lambda_B\times\begin{cases}\frac{1-\lambda_A}{N-\lambda_A}v\left(0\right) & \text{if } z_i=0\\ \frac{\lambda_A}{N}v\left(L\right)+\frac{1-\lambda_A}{N}v\left(0\right) & \text{if } z_i=L\\ v\left(L\right) & \text{if } z_i=2L.\end{cases} \tag{5}$$

Generally, the share price of each firm is its standalone value plus the expected takeover premium. If $z_i=2L$, the market maker of firm i knows for sure that the activist purchased L shares of the firm. Since the activist buys shares of firm i only if she identified it as the target, the market maker infers that firm i is indeed the target, and ascribes probability λ_B that it will receive a takeover offer. This explains the term behind $p_i\left(2L;\lambda_A,\lambda_B\right)$. By contrast, if $z_i=L$ then the market maker cannot distinguish between events in which firm i is a target and the activist bought L shares and events in which the demand comes from liquidity traders. Based on the prior, the market maker believes that firm i receives a takeover offer with probability $\frac{\lambda_B}{N}$. With probability λ_A, the activist owns L shares of the target, and the expected takeover premium is $v\left(L\right)$. With probability $1-\lambda_A$, the activist is not a shareholder of the target, and the expected takeover premium is $v\left(0\right)$. This explains the term behind $p_i\left(L;\lambda_A,\lambda_B\right)$. Finally, if $z_i=0$ then the market maker knows the activist did not buy shares in the firm either because firm i is not the target, which happens with probability $\lambda_A\frac{N-1}{N}$, or because she did not search, which happens with probability $1-\lambda_A$. The term behind $p_i\left(0;\lambda_A,\lambda_B\right)$ is the weighted average

[25] We implicitly assume that if $\lambda_A=0$ then the market maker's off-equilibrium beliefs when $z_i\notin\{0,L\}$ are that the activist identified firm i as the target and the excess demand stems from the activist.

of these events.

Notice that if $z_i > 0$ then the share price increases in λ_B and λ_A. A higher probability that the bidder searches for a target increases the likelihood of a takeover of firm i, and hence, the value of holding its shares. Similarly, a higher probability that the activist searches for a target increases the value of the share of firm i, since if a bidder arrives but the incumbent board refuses to relinquish control, the presence of the activist can facilitate the transaction.[26]

3.2.1 Activist's decision to search

According to Lemma 3, the activist does not trade without searching, and hence, if she does not search her expected payoff is zero. The activist's expected payoff net of the search cost and the price of buying L shares is given by

$$\Pi_A(c_A, \lambda_A, \lambda_B) = -c_A + L \times \max\left\{0\ ,\ \begin{array}{c} \frac{1}{2}\lambda_B[v(L) - \frac{\lambda_A}{N}v(L) - \frac{1-\lambda_A}{N}v(0)] \\ -\gamma q[1 - \lambda_B \int_{\min\{b,\rho(L)\}}^{\infty} dF(\Delta)] \end{array}\right\}. \tag{6}$$

The informational advantage of the activist stems from knowing which of the N firms is the target and the fact that she can pressure the target board to accept a future takeover bid. The former matters as long as $N \geq 2$, while the latter matters as long as $v(L) > v(0) \Leftrightarrow \rho(L) < b$. Both pieces of information are valuable if and only if the bidder offers to take over the target (which happens with probability λ_B) and the activist can camouflage her trade as driven by liquidity (which happens with probability $\frac{1}{2}$). This explains the first line in (6), which is also the difference between $q + \lambda_B v(L)$ and $\frac{1}{2}p_i(2L; \lambda_A, \lambda_B) + \frac{1}{2}p_i(L; \lambda_A, \lambda_B)$. The second line in (6) is the activist's expected disutility if the target remains independent. If γ is sufficiently large, the activist's expected profit from buying L shares of the target is negative, which explains why the term in the curly brackets is bounded from below by zero.

The activist searches for the target if and only if $\Pi_A(c_A, \lambda_A, \lambda_B) \geq 0$, where $\Pi_A(c_A, \lambda_A, \lambda_B)$ is a decreasing function of c_A. The next lemma follows directly from these observations.

Lemma 4 *In any equilibrium there is a unique threshold $c_A^* \geq 0$ such that the activist searches for the target if and only if $c_A \in [0, c_A^*]$, where either $\Pi_A(c_A^*, \lambda_A, \lambda_B) = 0$, or $c_A^* = 0$ and $\Pi_A(0, \lambda_A, \lambda_B) < 0$.*

[26] If $z_i = 0$, the share price increases with λ_B but decreases with λ_A, as $z_i = 0$ is a stronger signal that firm i is not a target the higher is the activist's search intensity.

Lemma 4 implies that in any equilibrium, $\lambda_A = H(c_A^*)$, where the threshold c_A^* is determined by the activist's indifference between searching and not searching, with the exception that if $\Pi_A(0, \lambda_A, \lambda_B) < 0$, the activist strictly prefers not searching even if search is costless, and hence, $c_A^* = 0$. This case occurs only if γ is sufficiently large.

Arbitrage activism Activist investors sometimes react to news on a deal by buying shares of the target with the objective of pressuring its board. In these situations, the activist buys shares *after* it becomes public that the company is a target. From the activist's perspective, she does not need to search for the target or speculate on the arrival of the bidder ($\lambda_B = 1$). This effect increases the activist's incentives to buy shares. On the other hand, the share price already reflects the information that the company is a target. This effect attenuates the activist's incentives to buy shares. Nevertheless, recall that buying shares of the target with the intent of challenging the board is still the activist's private information. Therefore, the activist can make a profit and affect corporate control outcomes even in these situations. Hereafter, we maintain the assumption that the activist has to search in order to identify the target and trades before the identity of the target becomes public information.

3.2.2 Bidder's decision to search

Without searching, the bidder does not know which firm is the target. Since the expected synergy from a takeover is negative, the bidder does not make an offer to any of the N firms and his expected payoff is zero. Suppose the bidder searched and identified firm i as the target, and the activist owns α shares in that firm. Based on the discussion that follows Proposition 1, the expected surplus from the takeover is

$$w(\alpha) = \int_{\min\{b,\rho(\alpha)\}}^{\infty} \Delta dF(\Delta), \tag{7}$$

and the expected takeover premium is $v(\alpha)$. The bidder's expected profit is the surplus generated by the takeover less the expected takeover premium and the search cost, and it is given by

$$\Pi_B(c_B, \lambda_A) = -c_B + \psi(\lambda_A), \tag{8}$$

where

$$\begin{aligned}\psi(\lambda_A) &= \lambda_A(w(L)-v(L))+(1-\lambda_A)(w(0)-v(0)) \\ &= (1-s)\left[\int_b^{\infty}(\Delta-b)\,dF(\Delta)+\lambda_A\int_{\min\{b,\rho(L)\}}^{b}\Delta dF(\Delta)\right].\end{aligned} \tag{9}$$

The bidder searches for a target if and only if $\Pi_B(c_B,\lambda_A)\geq 0$, where $\Pi_B(c_B,\lambda_A)$ is a decreasing function of c_B. The next result, which follows directly from these observations, implies that in any equilibrium $\lambda_B = G(c_B^*)$, where the threshold c_B^* is determined by the bidder's indifference between searching and not searching.

Lemma 5 *In any equilibrium there is a unique threshold $c_B^*\geq 0$ such that the bidder searches for the target if and only if $c_B\in[0,c_B^*]$, where $c_B^*=\psi(\lambda_A)$.*

3.3 Equilibrium

Proposition 2 *An equilibrium always exists. In any equilibrium, $c_B^*=\psi(H(c_A^*))>0$. If $c_A^*>0$ then c_A^* is given by the solution of $\mu(x)=0$, and if $c_A^*=0$ then $\mu(0)\leq 0$, where*

$$\mu(x)\equiv\Pi_A(x,H(x),G(\psi(H(x)))). \tag{10}$$

Moreover, there is $\overline{\gamma}>0$ such that if $\gamma\in[0,\overline{\gamma}]$ then $c_A^*>0$ in any equilibrium.

According to Lemma 2 and Lemma 3, if $b\leq\rho(L)$ then the activist's threat of running a proxy fight is not credible enough in equilibrium to relax the resistance of the incumbent board to the takeover. In this region, the activist does not affect the takeover process or the incentives of the bidder to search for a target. Nevertheless, the activist has incentives to become a shareholder of the target: Knowing which firm is likely to be a target gives the activist informational advantage that makes the purchase of the target shares a profitable investment. This information is valuable only if the bidder is likely make a takeover offer, and therefore, the gains from the speculative trade and the activist's incentives to search increase with the likelihood that the bidder searches for a target.[27] In other words, higher M&A activity increases the incentives of activist investors to speculate. In this region, the equilibrium is always unique.

[27] While the target share price increases with λ_B, the activist overall benefits from higher λ_B since higher λ_B also increases the value of her private information.

We name this region as the *"selection region"*, since the activist invests in firms that are likely to be targets, but her investment has no real effect.

If $b > \rho(L)$ and the activist is a shareholder of the target, she can pressure the incumbent board and relax its resistance to the takeover. Therefore, the bidder has stronger incentives to search for a target if the activist is expected to be a shareholder. This observation implies that the activist affects the takeover process even if ex-post her threat of running a proxy fight is not credible (i.e., $\Delta < \rho(L)$). In turn, and similar to the reasoning in the selection region, the activist's incentives to search increase with the bidder's search intensity. However, since here the activist affects the takeover process, her informational advantage is more significant, and consequently, the speculative gains are higher. Overall, higher intensity of shareholder activism (for example, as reflected by the number of schedule 13D filings) will increase the incentives of bidders to search for companies and approach them with takeover offers, and vice versa. We name this region as the *"treatment region"*, since the activist invests in firms that are likely to be targets, and by investing in these firms, not only she facilitates the takeover process once the offer is on the table, but she also provides bidders with stronger incentives to make the offer in the first place.

It follows that in the treatment region the game exhibits *strategic complementarity* in search decisions.[28] Strategic complementarity can lead to multiple equilibria. In Appendix C we show that multiple equilibria are likely to exist when the treatment region, measured by the length of the interval $[\rho(L), b]$, is large. The existence of multiple equilibria suggests that due to effective shareholder activism, the market for corporate assets can experience episodes of high volume of transactions ("hot markets") and episodes of low volume of transactions ("cold markets"), without any apparent change in the underlying fundamentals. That is, the activity in the market for corporate control is self-fulfilling and unpredictable.

3.4 Comparative statics

In this section we study the key comparative statics of the model. While the equilibrium may not be unique, all equilibria of the game can be ranked by the ex-ante probability that the

[28] Strategic complementarity arises when the best response functions are increasing. Since $\Pi_A(c_A, \lambda_A, \lambda_B)$ decreases with c_A and increases with λ_B, the activist's best response, c_A^*, increases in λ_B. Similarly, since $\Pi_B(c_B, \lambda_A)$ decreases with c_B and increases with λ_A, the bidder's best response, c_B^*, increases in λ_A.

target firm is acquired by the bidder,

$$\theta^* \equiv G\left(c_B^*\right)\left[\int_b^{\infty} dF\left(\Delta\right) + H\left(c_A^*\right)\int_{\min\{b,\rho(L)\}}^{b} dF\left(\Delta\right)\right]. \tag{11}$$

Indeed, c_B^* is an increasing function of c_A^*, and θ^* increases in both c_B^* and c_A^*. When multiple equilibria exist, we study the comparative statics of equilibria with the smallest and largest θ^*. We denote these two equilibria by $\underline{\theta}^*$ and $\overline{\theta}^*$, respectively. Focusing on extremal equilibria is common in games of strategic complementarities (e.g., Vives (2005)).[29]

Proposition 3 *Suppose $\gamma \in [0, \overline{\gamma}]$ and θ^* is either $\underline{\theta}^*$ or $\overline{\theta}^*$. Then:*

(*i*) *If $b \leq \rho\left(L\right)$ then θ^* does not change with L, κ or γ, and it decreases in b and s.*

(*ii*) *If $b > \rho\left(L\right)$ then θ^* increases in L, decreases in κ, and is ambiguous with respect to γ, b, and s.*

In the selection region where $b \leq \rho\left(L\right)$, the bidder's incentives to search are unaffected by the activist's presence. Therefore, θ^* does not change with parameters that only affect the incentives of the activist to intervene. By contrast, θ^* is decreasing with the incumbent board's private benefits and the target's bargaining power. Intuitively, with higher b or s, the bidder has to pay a higher price for the target. Lower profitability decreases the bidder's incentives to search, and thereby, the probability of a takeover.

In the treatment region where $b > \rho\left(L\right)$, the bidder's incentives to search are affected by the activist's presence, as her threat to run a proxy fight is credible. The credibility of this threat increases with L and decreases with κ. There are two effects. First, the bidder's incentives to search increase since reaching an acquisition agreement with the incumbent board is more likely. Second, the activist's private information of her being a shareholder of the target has a higher value, which increases her profits from speculative trades. Therefore, both the activist and the bidder have stronger incentives to search, thereby increasing θ^*. The effect of γ is more nuanced: Higher γ increases the credibility of the activist's threat since she has more to lose if the takeover fails, but the activist also suffers a larger disutility if the target

[29] Two technical comments are in place. First, we assume that the least and the greatest fixed points always exist. For example, if the function $\hat{\mu}\left(x\right) \equiv \mu\left(x\right) + x$ is monotonic, which holds for large N, then by Tarski's fixed point theorem, it has the least and the greatest fixed points. Second, we focus on local comparative statics, when the equilibrium continues to exist upon small changes in the parameter.

remains independent. The former effect increases the activist's incentives to search while the latter decreases these incentives. In Appendix C, we show that the former effect can dominate the latter. In those cases, short-termism reinforces the ability of activists to facilitate value-increasing takeovers.

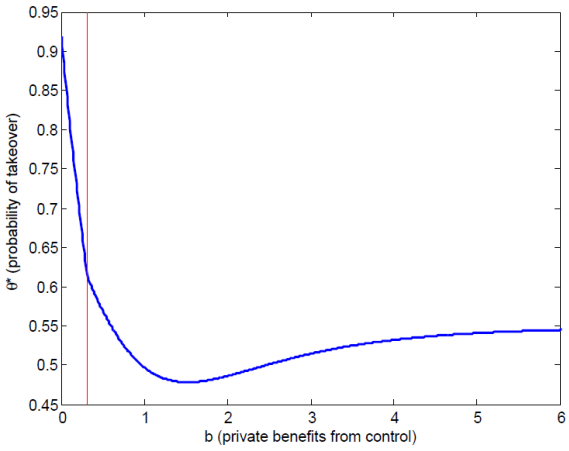


Figure 3 - The effect of b on θ^*

$L = 0.10, N = 100, \gamma = 0, s = 0.95, \kappa = 0.03, F(\Delta) = 1 - e^{-\Delta}, G(c) = H(c) = 1 - e^{-50c}$

Interestingly, as shown by Figure 3, θ^* can *increase* with b and s in the treatment region. All else being equal, higher b and s is likely to increase the takeover premium paid by the bidder. While the bidder's incentives to search decrease, the activist's incentives to search increase. Not only the activist expects a higher premium when the bidder arrives, but her threat of running a proxy fight becomes more credible (the interval $[\rho(L), b]$ expands). Since the bidder benefits from the activist's increased search effort, the indirect effect of b and s on the bidder's incentives to search can be positive. Due to the strategic complementarity, the overall probability of a takeover can increase. Therefore, contrary to the common wisdom, the probability of a takeover can increase with the resistance of the board to the takeover, as such resistance creates more investment opportunities for the activist.[30]

[30] In some cases, higher b and s can increase the bidder's incentives to search even if λ_A is held constant. Intuitively, if b is small, the bidder can reach an agreement even if the activist does not intervene, and the bidder pays a premium of $s\Delta + (1-s)b$. If b is large, the bidder can reach an agreement only if the activist intervenes, in which case, he pays a lower premium of $s\Delta$. The intuition for changes in s is slightly different. If s is small, the activist has no incentives to intervene and the bidder cannot reach an agreement with an entrenched board, resulting with a zero profit. If s is high, the activist has stronger incentives to intervene, and the bidder can acquire the target and make a profit of $(1-s)\Delta$.

Due to strategic complementarities in the treatment region, a small change in one of the parameters of the model can have an amplified effect on θ^*. For example, consider a change in regulation that eases the proxy access by decreasing κ. With an easier proxy access, the activist's threat to run a proxy fight is more credible. As a result, the bidder and the activist have stronger incentives to search since reaching an agreement between the bidder and incumbent is more likely. These two direct effects feedback due to strategic complementarities and result in a large indirect effect on the probability of a takeover. This logic also implies that polices that undermine shareholder activism but do not affect bidders directly (e.g., two-tier "anti-activism" poison pills) will still have a significant effect on takeovers.

3.4.1 Abnormal returns

The model provides a framework to study the abnormal returns around the announcements of 13D filings by activist investors and acquisition agreements. We assume that after trade takes place but before the arrival of the bidder becomes public, the activist must file a 13D schedule if and only if she is a shareholder of firm i. According to Lemma 3, if the activist files schedule 13D then the firm's share price jumps to $p_i\left(2L;\lambda_A^*,\lambda_B^*\right)$, and otherwise the price drops to $p_i\left(0;\lambda_A^*,\lambda_B^*\right)$. Below we derive the expressions for the average abnormal returns in our model.

Proposition 4 *The average abnormal returns around the announcement of event χ is positive and given by:*

$$AR\left(\chi\right)=\begin{cases}\frac{\lambda_B^*}{2}\left[v\left(L\right)-\frac{\lambda_A^*}{N}v\left(L\right)-\frac{1-\lambda_A^*}{N}v\left(0\right)\right] & \text{if } \chi \text{ is a 13D filing by an activist}\\ \left[\frac{1}{1-F(\min\{b,\rho(L)\})}-\lambda_B^*\right]v\left(L\right) & \text{if } \chi \text{ is an acquisition agreement preceded by a 13D filing}\\ \left[\frac{1}{1-F(b)}-\lambda_B^*\frac{1-\lambda_A^*}{N-\lambda_A^*}\right]v\left(0\right) & \text{if } \chi \text{ is an acquisition agreement not preceded by a 13D filing}\end{cases} \tag{12}$$

The average abnormal return around the announcement of a 13D filing is affected by parameters that govern the activist's incentives to intervene, but only in the treatment region. Perhaps surprisingly, it can decrease with L and γ, and increase with κ. Intuitively, these changes strengthen the ability of the activist to relax any opposition to the takeover. While

one might expect the market's reaction to a 13D filing to be stronger in these cases, the share price prior to the announcement already reflects the new expectations, and therefore, the overall (positive) reaction to a 13D filing can be smaller.

Proposition 4 also implies that the average abnormal return around the announcement of an acquisition agreement is *smaller* when it is preceded by a 13D filing than when it is not. There are two reasons. First, the price prior to the announcement on the takeover is already high if it is public information that the activist is a shareholder of the target. Second, conditional on the announcement of an acquisition agreement, the new price is higher if the activist is *not* a shareholder. Intuitively, without the pressure of the activist, the incumbent board agrees to a takeover only if the premium is sufficiently high to convince him to forgo the private benefits of control.

4 Extensions

4.1 Additional channels of complementarity

In this section, we highlight three different channels through which activist investors complement the effort of bidders to acquire companies.

4.1.1 Full commitment

Suppose the bidder can commit to act in the best interests of target shareholders after winning a proxy fight. Under this assumption, the bidder can credibly promise to pay target shareholders the "fair price", $q + s\Delta$, if he is given control of the board. Therefore, target shareholders are indifferent between giving control to the bidder and the activist, as in both cases the shareholder value is $q + s\Delta$. If $b \leq \Delta$, shareholders reelect the incumbent regardless of the identity of the rival team. If $\Delta < b$ and a proxy fight is initiated, the incumbent always loses. The bidder has incentives to run a proxy fight only if his expected payoff, $(1 - s)\,\Delta - \kappa$, is non-negative. The activist's incentives are the same as in Lemma 2 part (ii). Clearly, the bidder and the activist have no incentives to incur the costs and run a proxy fight if the other party is also expected to do so.

Lemma 6 *Suppose the first round of negotiations fails. Then:*

(*i*) *The bidder runs a proxy fight if and only if the activist does not run a proxy fight and*

$$\frac{\kappa}{1-s} \leq \Delta < b. \tag{13}$$

Whenever the bidder runs a proxy fight, she wins.

(*ii*) *If the activist owns α shares of the target, the activist runs a proxy fight if and only if the bidder does not run a proxy fight and*

$$\rho(\alpha) \leq \Delta < b. \tag{14}$$

Whenever the activist runs a proxy fight, she wins.

According to Lemma 6, the activist's threat of running a proxy fight is more credible than the bidder's if $\rho(\alpha) < \frac{\kappa}{1-s}$. The activist is likely to have stronger incentives than the bidder to run a proxy fight for three different reasons. First, since activists have more governance expertise due to their experience in challenging entrenched incumbents of other public companies (e.g., understanding the proxy solicitation process), they are likely to face lower costs of running a proxy fight than those faced by potential bidders. Second, short-termism ($\gamma > 0$) gives the activist stronger incentives to close a deal. In order to secure a quick exit on her investment, the activist would launch a costly proxy fight in circumstances that the bidder would not. Third, target shareholders typically extract most of the value that is created by the takeover (high s; for example, see Betton et al. (2008)), and hence, the activist has more to gain from running a proxy fight. Overall, if $\rho(\alpha) < \frac{\kappa}{1-s}$, the activist complements the bidder's effort to acquire the target even if the bidder can commit to act in the best interests of target shareholders.

Importantly, if $b \leq \frac{\kappa}{1-s}$ then the bidder does not run a proxy fight to replace the target board in any equilibrium of the subgame, and the analysis is identical to Section 3. In Section 3, the threat of running a proxy fight was not credible because target shareholders never elected the bidder's nominees to the board, while here the threat is not credible because the benefit from replacing the incumbent does not compensate the bidder for the cost of running a proxy fight. In Appendix B, we show that the results in Section 3 continue to hold qualitatively when $\frac{\kappa}{1-s} < b$. They key difference is that the region in which the bidder can reach an agreement with the incumbent board without the activist's pressure is expanded from $[b, \infty)$ to $[\frac{\kappa}{1-s}, \infty)$,

and the region in which the activist's pressure is necessary is scaled down from $[\rho(\alpha), b)$ to $[\rho(\alpha), \frac{\kappa}{1-s})$.

4.1.2 Influencing voting outcomes

Activists can help bidders overcome the resistance of incumbents to takeovers by voting their shares for the bidder's nominees and lobby other shareholders at the proxy fight stage. To emphasize this channel, suppose the bidder can commit to act in the best interests of target shareholders and the activist's threat of running a proxy fight is not credible. Our key assumption is that the likelihood that shareholders vote for the bidder's nominees at the proxy fight is higher when the activist is a shareholder of the target than when she is not. Specifically, suppose that if the bidder runs a proxy fight then with probability $1-\varepsilon(\alpha) \in (0,1)$ target shareholders vote rationally and with probability $\varepsilon(\alpha)$ they vote for the incumbent regardless of the circumstances. Intuitively, diffuse shareholders may abstain or vote blindly for the incumbent because of the false presumption that it is protecting their interests. Alternatively, some shareholders are biased toward management because of their business ties with the target (e.g., Cvijanovic et al. (2015)). Moreover, we assume that $\varepsilon(\cdot)$ is a decreasing function, and for simplicity, $\varepsilon(L) = 0$.

Under these assumptions, if the activist owns α share of the target, the bidder is facing a cost of $\frac{\kappa}{1-\varepsilon(\alpha)}$ per unit of success when running a proxy fight.[31] The analysis of the modified model is the same as in Section 4.1.1 when $b < \rho(L)$, with the exception that κ is replaced by $\frac{\kappa}{1-\varepsilon(L)}$. According to Lemma 6, if $\frac{\kappa}{1-s} < b \leq \frac{\kappa/(1-\varepsilon(L))}{1-s}$ then the bidder's threat of running a proxy fight is credible if and only if the activist is a shareholder of the target. With more credibility, the bidder can overcome the incumbent's resistance and acquire the target. Through this channel the activist can exercise influence even when her own threat of running a proxy fight is not credible.

4.1.3 Soliciting takeover bids

Activist investors have incentives to solicit bids once they invest in companies that they believe are good candidates for a takeover. Solicitation can involve meeting with potential bidders or announcing their intent to pressure management to sell the firm. In the context of our model,

[31] Suppose the bidder expects to win if all shareholders are rational (otherwise, she never runs a proxy fight). Also, let Π_{win} be the bidder's payoff if she wins the proxy fight and Π_{lose} if she loses. The bidder will run a proxy fight if and only if $(1-\varepsilon(\alpha))\Pi_{win} + \varepsilon(\alpha)\Pi_{lose} - \kappa > \Pi_{lose}$, which holds if and only if $\Pi_{win} - \Pi_{lose} > \frac{\kappa}{1-\varepsilon(\alpha)}$.

suppose the bidder observes the activist's decision to search and the firm in which she invested before making his own decision to search (for example, by following the filing of a 13D schedule by the activist). This modification does not change the analysis of the takeover negotiations and proxy fights phase. However, it changes the search and position building phase. Since the search decisions are now made sequentially, the equilibrium is unique. As in the baseline model, if the activist does not search, she does not buy shares of any firm, and the bidder's problem is the same as in the baseline model, where $\lambda_A = 0$. However, if the activist searched and became the shareholder of firm i, the bidder would infer that firm i is the target and start negotiating a takeover with its board. The decisions of the activist to search and invest in firm i not only inform the bidder that the takeover of firm i can create value (this effect exists even in the selection region), but they also reassure the bidder that he will face a weaker opposition to the takeover if the offer is fair (this effect exists only in the treatment region). We conclude that solicitation is another channel through which activists affect corporate control outcomes.

4.2 Channels of substitution

In this section, we highlight two channels through which activist investors compete away the bidder's rent from a takeover.

4.2.1 Incumbent boards as motivated sellers

A key friction behind the analysis in Sections 3 is the reluctance of incumbents to relinquish control. However, in management buyouts or when bidders promise incumbents large bonuses or future employment if the takeover succeeds (Grinstein and Hribar (2004) and Hartzell et al. (2004)), the incumbents may be too motivated to sell the firm. If there is a concern that the interests of target shareholders are compromised, activist investors will challenge the deal with the intent of either blocking it or "forcing" the bidder to sweeten the bid (Jiang et al. (2015)).[32]

To stress this point, suppose that unless forced otherwise, the incumbent board would sell the firm for a zero premium. Unlike the incumbent, the activist, if given control, would negotiate the fair price, $q + s\Delta$. Therefore, target shareholders always elect the activist to the

[32] For example, during the management buyout of Dell in 2013, the pressure of the activist investor Carl Icahn resulted in the increase of the offer price. See businessinsider.com, "Michael Dell Sweetens His $25 Billion Offer; Icahn Vows To Fight On", 8/3/2013.

board whenever she runs a proxy fight. As in Section 3, the activist has incentives to run a proxy fight if and only if $\rho(\alpha) < \Delta$. It follows that the target is always acquired by the bidder: If the activist owns α shares of the target and $\rho(\alpha) < \Delta$ then the bidder pays $q + s\Delta$, and in all other cases the bidder pays q. The analysis of the search and position building phase is similar with the exception that $v(\alpha)$ is replaced by $\int_{\rho(\alpha)}^{\infty} s\Delta dF(\Delta)$ and $w(\alpha)$ is replaced by $\int_0^{\infty} \Delta dF(\Delta)$. While the activist's incentives to search increase with λ_B, the bidder's incentives to search *decrease* with λ_A. Intuitively, the activist increases the expected takeover premium that the bidder is required to pay, without increasing the likelihood that the incumbent board agrees to sell the firm. Since the acquisition is less profitable, the bidder has fewer incentives to search for the target when the activist is likely to be a shareholder.

The activist benefits target shareholders by increasing the takeover premium, but at the same time, the activist harms them indirectly by disincentivizing the bidder to search and make a takeover offer. Nevertheless, when the incumbent board is motivated to sell the firm, he might actively solicit bids, and therefore, the bidder's search friction is likely to be second order. For example, in a management buyout, a search by the bidder is not needed at all and the activist would generally benefit target shareholders by "forcing" the bidder to sweeten the bid.

4.2.2 Increasing the target standalone value

Activist investors may also have the expertise to propose and execute operational, financial, and governance related policies that increase the standalone value of the firm. To study how this additional expertise interacts with corporate control activism, we modify the baseline model by assuming that if the target remains independent and the activist's proposal is implemented then the incumbent board loses his private benefits of control but the target's standalone value instantly increases by $\zeta\Delta$, where $\zeta \in (0, 1]$. We assume that without the activist, the incumbent board is either unaware or does not have the expertise to implement this proposal.[33] The baseline model is a special case where $\zeta = 0$.

The activist's ability to increase the standalone value of the target affects the bidder in two different ways. First, since the target standalone value is now higher when the activist is a shareholder, the bidder may have to pay a higher premium, which reduces her incentives to search. Second, because the bidder is expected to pay a higher premium, the activist has

[33] When $\zeta\Delta < b$ the incumbent does not implement the proposal voluntarily, and the activist's intervention can be interpreted as the removal of inefficiencies caused by the incumbent's consumption of private benefits.

stronger incentives to intervene if the incumbent board rejects the takeover offer or refuses to implement the proposal. This effect increases the bidder's incentives to search. Generally, depending on the severity of the agency friction in the target firm, the expertise of the activist can either attenuate or amplify the complementarity between the bidder's and the activist's search efforts. Specifically, in Appendix B we show that as long as $\zeta < 1$ the second effect dominates if b is sufficiently large. Intuitively, in those cases, the bidder cannot acquire the target unless the activist relaxes the incumbent's resistance. By contrast, as long as $\zeta > 0$ the first effect dominates if b is sufficiently small. Intuitively, in those cases the bidder is likely to reach an agreement even without having the activist pressuring the incumbent board. Not only the activist's presence does not improve the likelihood of reaching an agreement, but it also forces the bidder to pay a higher premium.

The capacity to acquire

If the activist has the ability to increase the standalone value of the target and make a takeover bid, then similar to the bidder in our model they could suffer from a commitment problem as discussed in Section 3.1. However, there is crucial difference between the bidder and the activist. Unlike the bidder, the activist does not have to acquire more than 50% of the target and take it private in order to create value; she can increase the standalone value of the target even if its ownership structure does not change. Therefore, while the activist may be tempted to low-ball the takeover offer once she gets control of the target board, these attempts are doomed to fail since target shareholders know that if they reject the offer, the activist will inevitably implement the value-increasing proposal in order to maximize the value of her own stake in the target.[34] Essentially, unlike the bidder who can add value only through the takeover, the activist cannot commit not to increase value if the takeover offer is rejected by shareholders. Therefore, shareholders would not fear electing the activist to the board, even if the activist has the capacity to acquire. In this respect, activists are more resilient than bidders to the commitment problem in takeovers.

[34] We implicitly assume that the activist has enough incentives to implement the value-increasing proposal even if she owns less than 50% of the target. Otherwise, there is no difference between the activist and the bidder in our model.

5 Conclusion

This paper studies the role of activist investors in the market for corporate control. We focus on two key frictions: agency problems in public corporations that result in excessive resistance of incumbents to takeovers, and the costly search for corporate assets with which synergies can be created. Unlike bidders, activists are on the same side of the negotiating table as other shareholders of the target, and hence, enjoy higher credibility when campaigning against the incumbents. Building on this insight, our analysis demonstrates that although both bidders and activists can use similar techniques to challenge corporate boards (i.e., proxy fights), activists are more effective in relaxing their resistance to takeovers. In this respect, activist investors complement the effort of bidders to acquire companies by making these companies available for sale. Since bidders search for targets only if they believe that these companies are also available for sale, there is strategic complementarity between the search of activists for firms that are likely to receive a takeover bid and the search of bidders for targets with which they can create synergies. Combined, the analysis sheds light on the interaction between M&A and shareholder activism and provides a framework to identify the treatment and the selection effects of shareholder activism.

References

[1] Agrawal, Anup, and Ralph Walkling, 1994, Executive careers and compensation surrounding takeover bids, *Journal of Finance* 49, 985-1014.

[2] Atanasov, Vladimir A., Audra L. Boone, and David Haushalter, 2010, Is there shareholder expropriation in the United States? An analysis of publicly traded subsidiaries, *Journal of Financial and Quantitative Analysis* 45, 1-26.

[3] Atanasov, Vladimir A., Bernard S. Black, and Conrad S. Ciccotello, 2014, Unbundling and measuring tunneling, *University of Illinois Law Review* 5, 1697-1738.

[4] Bagnoli, Mark, Roger Gordon, and Barton L. Lipman, 1989, Stock repurchase as a takeover defense, *Review of Financial Studies* 2, 423-443.

[5] Baron, David P., 1983, Tender offers and management resistance, *Journal of Finance* 38, 331-343.

[6] Bates, Thomas W., Michael L. Lemmon, and James S. Linck, 2006, Shareholder wealth effects and bid negotiation in freeze-out deals: Are minority shareholders left out in the cold?, *Journal of Financial Economics* 81, 681-708.

[7] Bebchuk, Lucian A., and Oliver D. Hart, 2001, Takeover bids vs. proxy fights in contests for corporate control, Working paper.

[8] Bebchuk, Lucian A., John Coates IV, and Guhan Subramanian, 2002, The powerful antitakeover force of staggered boards: Theory, evidence, and policy, *Stanford Law Review* 54, 887-951.

[9] Becht, Marco, Patrick Bolton, and Ailsa Röell, 2003, Corporate governance and control, in George Constantinides, Milton Harris, and René Stulz (eds.), *The Handbook of the Economics of Finance* (Amsterdam: North-Holland).

[10] Becht, Marco, Julian R. Franks, Jeremy Grant, and Hannes F. Wagner, 2015, The returns to hedge fund activism: An international study, Working paper.

[11] Berle, Adolf A., and Gardiner C. Means, 1932, *The Modern Corporation and Private Property* (New York: Macmillan).

[12] Bhattacharya, Utpal, 1997, Communication costs, information acquisition, and voting decisions, *The Review of Financial Studies* 10, 1065-1097.

[13] Berkovitch, Elazar, and Naveen Khanna, 1990, How target shareholders benefit from value-reducing defensive strategies in takeovers, *Journal of Finance* 45, 137-156.

[14] Betton, Sandra, B. Espen Eckbo, and Karin S. Thorburn, 2008, Corporate Takeovers, in Eckbo, B.E. (ed.), *Handbook of Corporate Finance: Empirical Corporate Finance*, vol. 2, pp. 291-430. chap. 15.

[15] Brav, Alon, Wei Jiang, Frank Partnoy, and Randall Thomas, 2008, Hedge fund activism, corporate governance, and firm performance, *Journal of Finance* 63, 1729-1775.

[16] Bulow, Jeremy, Ming Huang, and Paul Klemperer, 1999, Toeholds and takeovers, *Journal of Political Economy* 107, 427-454.

[17] Burkart, Mike, 1995, Initial shareholdings and overbidding in takeover contests, *Journal of Finance* 50, 1491-1515.

[18] Burkart, Mike, Denis Gromb, and Fausto Panunzi, 2000, Agency conflicts in public and negotiated transfers of corporate control, *Journal of Finance* 55, 647-677.

[19] Burkart, Mike, and Samuel Lee, 2015, Hedge fund activism vs. hostile takeover bids, Working paper.

[20] Cornelli, Francesca, and David Li, 2002, Risk arbitrage in takeovers, *The Review of Financial Studies* 15, 837-868.

[21] Cvijanovic Dragana, Amil Dasgupta, and Kostas Zachariadis, Ties that bind: How business connections influence mutual fund activism, *Journal of Finance,* forthcoming.

[22] Edmans, Alex, 2014, Blockholders and corporate governance, *Annual Review of Financial Economics* 6, 23-50.

[23] Fos, Vyacheslav, 2015, The disciplinary effects of proxy contests, *Management Science*, forthcoming.

[24] Gilson, Ronald J., 2001, Unocal fifteen years later (and what we can do about it), *Delaware Journal of Corporate Law* 26, 491-513.

[25] Gilson, Ronald J., and Alan Schwartz, 2001, Sales and elections as methods for transferring corporate control, *Theoretical Inquiries in Law* 2, 783-814.

[26] Gomes, Armando, 2012, Takeovers, freezeouts, and risk arbitrage, Working paper.

[27] Gordon, Elizabeth A., Elaine Henry, and Darius Palia, 2004, Related party transactions and corporate governance, *Advances in Financial Economics* 9, 1-27.

[28] Greenwood, Robin, and Michael Schor, 2009, Investor activism and takeovers, *Journal of Financial Economics* 92, 362-375.

[29] Grinstein, Yaniv, and Paul Hribar, 2004, CEO compensation and incentives: Evidence from M&A bonuses, *Journal of Financial Economics* 73, 119-143.

[30] Grossman, Sanford J., and Oliver D. Hart, 1980, Takeover bids, the free-rider problem, and the theory of the corporation, *Bell Journal of Economics* 11, 42-64.

[31] Harford, Jarrad, Dirk Jenter, and Kai Li, 2011, Institutional cross-holdings and their effect on acquisition decisions, *Journal of Financial Economics* 99, 27-39.

[32] Harris, Milton, and Artur Raviv, 1988, Corporate governance: Voting rights and majority rules, *Journal of Financial Economics* 20, 203-235.

[33] Hartzell, Jay C., Eli Ofek, and David Yermack, 2004, What's in it for me? CEOs whose firms are acquired, *Review of Financial Studies* 17, 37-61.

[34] Hirshleifer, David, and Sheridan Titman, 1990, Share tendering strategies and the success of hostile takeover bids, *Journal of Political Economy* 98, 295-324.

[35] Jenter, Dirk, and Katharina Lewellen, 2015, CEO preferences and acquisitions, *Journal of Finance*, forthcoming.

[36] Jiang, Wei, Tao Lei, and Danqing Mei, 2015, Influencing control: Jawboning in risk arbitrage, Working paper.

[37] Kyle, Albert S., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315-1335.

[38] Kyle, Albert S., and Jean-Luc Vila, 1991, Noise trading and takeovers, *Rand Journal of Economics* 22, 54-71.

[39] Martin, Kenneth, and John McConnell, 1991, Corporate performance, corporate takeovers, and management turnover, *Journal of Finance* 46, 671-687.

[40] Matvos, Gregor, and Michael Ostrovsky, 2008, Cross-ownership, returns, and voting in mergers, *Journal of Financial Economics* 89, 391-403.

[41] Maug, Ernst, 1998, Large shareholders as monitors: Is there a trade-off between liquidity and control?, *Journal of Finance* 53, 65-98

[42] Maug, Ernst, 1999, How effective is proxy voting? Information aggregation and conflict resolution in corporate voting contests, Working paper.

[43] Mulherin, J. Harold, and Annette B. Poulsen, 1998, Proxy contests and corporate change: Implications for shareholder wealth, *Journal of Financial Economics* 47, 279-313.

[44] Ofer, Aharon R., and Anjan V. Thakor, 1987, A theory of stock price response to alternative corporate disbursement methods: Stock repurchases and dividends, *Journal of Finance* 42, 365-394.

[45] Rubinstein, Ariel, 1982, Perfect equilibrium in a bargaining model, *Econometrica* 50, 97-109.

[46] Shleifer, Andrei, and Robert W. Vishny, 1986, Large shareholders and corporate control, *Journal of Political Economy* 94, 461-488.

[47] Singh, Rajdeep, 1998, Takeover bidding with toeholds: The case of the owner's curse, *Review of Financial Studies* 11, 679-704.

[48] Vives, Xavier, 2005, Complementarities and games: New developments, *Journal of Economic Literature* 43, 437-479.

[49] Walkling, Ralph, and Michael Long, 1984, Agency theory, managerial welfare, and takeover bid resistance, *RAND Journal of Economics* 15, 54-68.

[50] Wulf, Julie, and Harbir Singh, 2011, How do acquirers retain successful target CEOs? The role of governance, *Management Science* 57, 2101-2114.

[51] Yılmaz, Bilge, 1999, Strategic voting and proxy contests, Working paper.

A Proofs of Section 3

Proof of Lemma 1. Generally, there are three scenarios to consider. The scenarios differ with respect to the composition of the target board after the proxy fight stage. Under all scenarios, target shareholders approve the acquisition agreement if it is brought to a shareholder vote if and only if the takeover offer is higher than the standalone value of the firm, q. Moreover, the bidder will not agree to pay more than $q + \Delta$ for the firm.

In the first scenario, the incumbent board is reelected and retains control of the target. The incumbent board would agree to sell the firm if and only if the bidder offers at least $q + b$ per share. Therefore, if $\Delta < b$ no agreement is reached and the target remains independent under the control of the incumbent. If $\Delta \geq b$ then the incumbent board and the bidder reach an agreement in which the expected takeover premium is $s\Delta + (1-s)b$: with probability $1-s$ the bidder proposes to pay $q+b$, which is the lowest price that is acceptable by both the incumbent board and the shareholders, and with probability s the incumbent board propose to receive $q + \Delta$, which is the highest price that the bidder would agree pay for the firm.

In the second scenario, the activist wins the proxy fight and controls the target board. If no agreement is reached with the bidder, the target remains independent, and the activist's payoff per share is $q - \gamma q$, which is the discounted long-term value of the target as a standalone firm. Therefore, the activist would agree to sell the firm if and only if the offer is higher than $q - \gamma q$. Since $\Delta > 0$, the bidder and the activist always reach an acquisition agreement that is also acceptable to target shareholders. With probability $1 - s$ the bidder offers q, which is the lowest price that is acceptable by both the activist and the shareholders, and with probability s the activist offers $q + \Delta$, which is the highest price the bidder would pay for the firm.

In the third scenario, the bidder wins the proxy fight and controls the target board. The argument is given in the main text. ■

Proof of Proposition 1. We start by proving that if $b \leq \Delta$ the bidder pays $q + s\Delta + (1-s)b$ and takes over the target after the first round of negotiations. Suppose $b \leq \Delta$. Based on Lemma 2, the activist will not run a proxy fight if the first round of negotiations fails. Since $b \leq \Delta$, all players expect the takeover to consume in the second round of negotiations, where the price is $q + s\Delta + (1-s)b$. Therefore, in the first round of negotiations, the incumbent board will reject any offer which is lower, and the bidder will reject any offer which is higher. If there are arbitrarily small waiting costs to either the bidder or the incumbent board, the deal will close in the first round.

Second, we prove that if $\rho(\alpha) \leq \Delta < b$ and the activist owns α shares of the target, the

bidder pays $q + s\Delta$ and takes over the target after the first round of negotiations. Based on Lemma 2, if the activist owns α shares of the target and $\rho(\alpha) \leq \Delta < b$, then shareholders would support the activist at the proxy fight if the first round of negotiations fails. Based on Lemma 1, all players expect that once the activist obtains control of the board, she will reach a sale agreement in which the bidder pays in expectations $q + s\Delta$ per share. The bidder realizes that any lower offer will be rejected by shareholders, who expect the activist to negotiate a higher offer at the second round. The bidder can afford to pay $q + s\Delta$. The bidder will not pay more than $q + s\Delta$, since he always has the option to pay that much in the second round when he negotiates with the activist. The incumbent board understands the bidder's incentives. The board also realizes that the takeover of the target is inevitable, and he will lose his private benefits of control. However, by accepting the offer $q + s\Delta$ the board can avoid the costly proxy fight. Therefore, the incumbent and the bidder reach an agreement in the first round where the offer is $q + s\Delta$, as required.

Last, we prove that in all other cases, the target remains independent under the incumbent board's control. According to Lemma 2, in all other cases, neither the bidder nor the activist initiate a proxy fight if the first round of negotiations fails. Therefore, the incumbent board retains control. Since in this region $\Delta < b$, based on Lemma 1, the incumbent board and the bidder will not reach an agreement in the second round of negotiations. Therefore, in the first round of negotiations, the incumbent board will reject any offer lower than $q + b$, and the bidder will reject any offer higher than $q + \Delta$. Thus, the parties will not reach an agreement in the first round as well, and the target remains independent.

The proof is completed by noting that (4) is average of these three cases and is a decreasing function of $\rho(\alpha)$, and that $\rho(\alpha)$ is a decreasing function of α. ■

Proof of Lemma 3. In the proof of this result we assume that if $\gamma = 0$ and the activist is indifferent between buying and not buying shares of firm i, the activist does not buy these shares. Without this assumption, equilibria in which the activist buys shares when indifferent would not survive perturbations such as arbitrarily small transaction costs.

Suppose it is a common knowledge that the activist owns $\alpha \geq 0$ shares of firm i and the bidder is expected to make a takeover offer to firm i with probability $\lambda \in [0, 1]$. Based on Propositions 1 and 5, the expected value of the firm (to all shareholders other than the activist) is $V(\alpha, \lambda) = q + \lambda v(\alpha)$. Note that $V(\alpha, \lambda)$ is strictly increasing in λ. Moreover, $\alpha > 0 \Rightarrow V(\alpha, \lambda) \geq V(0, \lambda)$. The expected value of each share for the activist is given by

$\Lambda(\alpha,\lambda) = V(\alpha,\lambda) - \Gamma(\alpha,\lambda)$, where

$$\Gamma(\alpha,\lambda) = \gamma q\left[1-\lambda+\lambda\int_0^{\min\{b,\rho(\alpha)\}} dF(\Delta)\right]. \tag{15}$$

Note that $\Gamma(\alpha,\lambda)$ is decreasing in λ. Therefore, $\Lambda(\alpha,\lambda)$ is increasing in λ.

We denote by α^* the number of shares of firm i the activist buys in equilibrium if she identifies firm i as a target, and by α_0 the number of shares of firm i she buys in equilibrium if she does not search. We prove the lemma in several steps.

First, suppose the activist searches and finds that firm i is *not* the target. We argue that the activist buys no shares. Indeed, the activist knows for sure that the bidder will not make an offer to this firm, and the shareholders will never support her at the proxy fight if no bidder has arrived. Therefore, the value of the firm from the activist's perspective is $q(1-\gamma)$. Since the market maker of firm i does not know for sure that firm i is not a target, he believes that there is a non-negative probability that firm i is the target when he observes a positive order-flow. Therefore, if $z_i > 0$ then $p(z_i) \geq q \geq q(1-\gamma)$, and the activist's expected profit is non-positive. For this reason, the activist never buys a stake in a firm she identifies not to be a target.

Second, we argue that if $\alpha_0 > 0$ then $\alpha^* > 0$. Suppose on the contrary, $\alpha_0 > 0$ and $\alpha^* = 0$. Let $p(\alpha)$ be the price the activist expects when she submits an order to buy α shares of firm i. Since $\alpha_0 > 0$, by revealed preferences, the activist's expected profit is positive if she buys α_0 shares of firm i without searching. In this case, the activist believes that each firm is equally likely to be the target, and therefore, the bidder makes a takeover offer to firm i with probability λ_B/N. Therefore, $\Lambda(\alpha_0,\lambda_B/N) - p(\alpha_0) \geq 0$. However, if the activist identifies firm i as the target, she expects the bidder to make the firm a takeover offer with probability $\lambda_B > \lambda_B/N$. Since $\Lambda(\alpha,\lambda)$ is strictly increasing in λ, $\Lambda(\alpha_0,\lambda_B) - p(\alpha_0) > 0$. This creates a contradiction, since the activist can make a strictly positive payoff from submitting order to buy $\alpha_0 > 0$ when she identifies firm i as a target.

Third, we argue that if $\alpha_0 > 0$ then $\alpha^* = \alpha_0$. Suppose on the contrary that $\alpha_0 > 0$ and $\alpha^* \neq \alpha_0$, and recall that $\alpha_0 > 0 \Rightarrow \alpha^* > 0$. First we argue that $\alpha^* = L$. Suppose on the contrary $\alpha^* \neq L$. If the activist buys α^* shares of firm i, then the market maker knows for sure that the activist demanded these shares. Since $\alpha^* \neq \alpha_0$, in equilibrium, the market maker infers that the activist identified firm i as a target, and hence, the bidder will make a takeover offer to firm i with probability λ_B. Therefore, $p(\alpha^*) = p(\alpha^* + L) = V(\alpha^*,\lambda_B)$. Since $\Lambda(\alpha^*,\lambda_B) \leq V(\alpha^*,\lambda_B)$, the activist is better off not buying α^* shares,

creating a contradiction.[35] Second, note that if $\alpha_0 \neq \alpha^* = L$ then $\alpha_0 \neq L$. If the activist buys α_0 shares of firm i, then the market maker knows for sure that the activist demanded these shares. Since $\alpha_0 \neq \alpha^* = L$, in equilibrium the market maker infers that the activist did not search, and hence, the bidder will make a takeover offer to firm i with probability λ_B/N. Therefore, $p(\alpha_0) = p(\alpha_0 + L) = V(\alpha_0, \lambda_B/N)$. Since $\Lambda(\alpha_0, \lambda_B/N) \leq V(\alpha_0, \lambda_B/N)$, the activist is better off not buying α^* shares, creating a contradiction. Overall, if $\alpha^* \neq \alpha_0$, $\alpha_0 > 0$, and $\alpha^* > 0$, then it must be $\alpha^* = \alpha_0 = L$, yielding a contradiction.

Fourth, we argue that $\alpha_0 = 0$. Suppose on the contrary $\alpha_0 > 0$. The previous argument implies $\alpha_0 = \alpha^*$. Suppose the activist does not search. Without searching, the expected value for the activist from each share is $\Lambda(\alpha^*, \lambda_B/N)$. Consider two cases. First, suppose $\alpha^* \neq L$. The market maker of firm i knows for sure that the activist bought α^* shares in firm i. Conditional on this event, the market maker believes that with probability λ_A/N the activist bought firm i because she identified it as the target, and with probability $1 - \lambda_A$ the activist bought firm i without searching and knowing it is the target. Combined, the market maker believes that the probability that the target receives a takeover offer is

$$h(\lambda_B) \equiv \frac{(1-\lambda_A)\frac{\lambda_B}{N} + \frac{\lambda_A}{N}\lambda_B}{(1-\lambda_A) + \frac{\lambda_A}{N}}. \tag{16}$$

Therefore, $p(\alpha^*) = p(\alpha^* + L) = V(\alpha^*, h(\lambda_B))$. Since $N \geq 2$ implies $h(\lambda_B) > \lambda_B/N$, $V(\alpha^*, h(\lambda_B)) > V(\alpha^*, \lambda_B/N)$. Since $\Lambda(\alpha^*, \lambda_B/N) \leq V(\alpha^*, \lambda_B/N)$, $\Lambda(\alpha^*, \lambda_B/N) < V(\alpha^*, h(\lambda_B))$, and the activist's expected profit is negative, creating a contradiction. Second, suppose $\alpha^* = L$. If $z_i = 2L$ then the market maker of firm i knows for sure that the activist bought L shares in firm i. For the same reasons as in the case where $\alpha^* \neq L$, it must be $p_i(2L) = V(L, h(\lambda_B))$. If $z_i = L$ there are three events the market maker considers:

1. With probability $\frac{1}{2}\lambda_A\frac{N-1}{N}$ the activist did not buy a stake because she searched and found that firm i is not the target. In this case, firm value is $V(0,0) = q$.

2. With probability $\frac{1}{2}\lambda_A\frac{1}{N}$ the activist bought a stake L since she identified firm i as the target. In this case, firm value is $V(L, \lambda_B)$.

3. With probability $\frac{1}{2}(1 - \lambda_A)$ the activist bought a stake L in firm i even though she did not search. In this case, firm value is $V(L, \lambda_B/N)$.

[35] Note that here and in fifth step below we invoke the assumption that if the activist is indifferent she does not buy any shares (note that this assumption is necessary only in the knife edge case where $\gamma = 0$).

Therefore, the share price is given by

$$p_i(L) = \lambda_A \tfrac{N-1}{N} V(0,0) + \lambda_A \tfrac{1}{N} V(L, \lambda_B) + (1-\lambda_A) V(L, \lambda_B/N). \tag{17}$$

Recall that $V(\alpha, \lambda)$ is linear in λ and equal to $q + \lambda v(\alpha)$. Therefore, the activist's expected profit is negative:

$$\begin{aligned} & \Lambda(L, \lambda_B/N) - \tfrac{1}{2} p_i(2L) - \tfrac{1}{2} p_i(L) \\ = \; & V(L, \lambda_B/N) - \Gamma(L, \lambda_B/N) - \tfrac{1}{2} V(L, h(\lambda_B)) - \tfrac{1}{2} \left[\begin{array}{c} \lambda_A \tfrac{N-1}{N} V(0,0) + \lambda_A \tfrac{1}{N} V(L, \lambda_B) \\ + (1-\lambda_A) V(L, \lambda_B/N) \end{array} \right] \\ = \; & -\tfrac{1}{2} \tfrac{\lambda_A \lambda_B}{N} \tfrac{N-1}{(1-\lambda_A)N + \lambda_A} v(L) - \Gamma(L, \lambda_B/N) \; \leq 0. \end{aligned}$$

This creates a contradiction. We conclude that the activist does not buy a positive stake of any firm unless she identifies the firm as the target.

Fifth, we show that if $\alpha^* > 0$ then $\alpha^* = L$. Indeed, if $\alpha^* > 0$ but $\alpha^* \neq L$ then the market maker of firm i knows for sure that the activist bought α^* shares in firm i, and that firm i was identified as the target. Therefore, the share price is $V(\alpha^*, \lambda_B)$, while the activist value per share is $\Lambda(\alpha^*, \lambda_B) \leq V(\alpha^*, \lambda_B)$. The activist makes a non-positive profit and hence she is better off not buying a share, yielding a contradiction.

Finally, in any equilibrium, if the activist does not plan on buying shares of the firm she identifies as a potential target, then the activist has no incentives to search for a target in the first place. If $\alpha^* = L$ then $z_i \in \{L, 2L\}$. If $z_i = 2L$ then the market maker of firm i knows for sure that the activist bought L shares in firm i, which is identified by the activist as a potential target. Therefore, the probability that the bidder will make a takeover offer is λ_B, and $p_i(2L) = V(L, \lambda_B)$. Implicitly, we assume that if $\lambda_A = 0$ then the market maker's off-equilibrium beliefs when $z_i \neq L$ are that the activist bought L shares, and the activist identified the firm as a target. Under these beliefs, the price is $V(L, \lambda_B)$. If $z_i = L$ there are three events the market maker considers:

1. With probability $\frac{1}{2}\lambda_A \frac{N-1}{N}$ the activist did not buy a stake because she searched and found that firm i is not the target. In this case, firm value is $V(0,0) = q$.

2. With probability $\frac{1}{2}\lambda_A \frac{1}{N}$ the activist bought a stake L since she identified firm i as the target. In this case, firm value is $V(L, \lambda_B)$.

3. With probability $\frac{1}{2}(1-\lambda_A)$ the activist did not buy a stake because she did not search

for the target. In this case, firm value is $V(0, \lambda_B/N)$.

Combined,

$$\begin{aligned} p_i(L) &= \lambda_A \tfrac{N-1}{N} V(0,0) + \lambda_A \tfrac{1}{N} V(L, \lambda_B) + (1-\lambda_A) V(0, \lambda_B/N) \\ &= q + \lambda_B \tfrac{\lambda_A v(L) + (1-\lambda_A) v(0)}{N} \end{aligned} \tag{18}$$

as required. Note that if $z_i = 0$ either case 1 above or case 3 above can take place. Therefore,

$$p_i(0) = \tfrac{\lambda_A \frac{N-1}{N} V(0,0) + (1-\lambda_A) V(0, \lambda_B/N)}{\lambda_A \frac{N-1}{N} + (1-\lambda_A)} = q + \tfrac{1-\lambda_A}{N-\lambda_A} \lambda_B v(0), \tag{19}$$

as required. ■

Proof of Proposition 2. Note that $\mu(x)$ is continuous in x and $\lim_{x\to\infty} \mu(x) = -\infty$. Based on Lemma 4, $\lambda_A = H(c_A^*)$ in any equilibrium, and based on Lemma 5, c_B^* is uniquely determined by c_A^* and is given by $\psi(H(c_A^*))$. Moreover, based on (9) and since $b < \infty$, $c_B^* = \psi(H(c_A^*)) > 0$ in any equilibrium.

We prove that an equilibrium always exists. We consider two cases. First, suppose $\mu(x) < 0$ for all $x \geq 0$. We argue that $(c_A^*, c_B^*) = (0, \psi(0))$ is the unique equilibrium. Suppose on the contrary an equilibrium with $c_A^* > 0$ exists. In this case, $c_B^* = \psi(H(c_A^*))$, and the activist's profit when his cost is c_A^* is $\mu(c_A^*)$. Since $\mu(x) < 0$ for all $x \geq 0$, the activist strictly prefers not searching when $c_A = c_A^*$, contradicting the optimality of the threshold strategy c_A^*. Suppose $c_A^* = 0$. In this case, $\lambda_A^* = 0$, $c_B^* = \psi(0)$, and $\lambda_B^* = G(\psi(0))$. The activist's profit is $\Pi_A(c_A, \lambda_A^*, \lambda_B^*)$. Note that $\Pi_A(0, \lambda_A^*, \lambda_B^*) = \mu(0)$. Since $\mu(0) < 0$ and $\Pi_A(c_A, \lambda_A^*, \lambda_B^*)$ is decreasing c_A, $\Pi_A(c_A, \lambda_A^*, \lambda_B^*) < 0$ for all $c_A \geq 0$. It follows that the activist never searches and $c_A^* = 0$ is her optimal response. We conclude, if $\mu(x) < 0$ for all $x \geq 0$ then $(c_A^*, c_B^*) = (0, \psi(0))$ is the unique equilibrium. Second, suppose there is $\hat{x} \geq 0$ such that $\mu(\hat{x}) \geq 0$. Since $\lim_{x\to\infty} \mu(x) = -\infty$, by the intermediate value theorem the equation $\mu(x) = 0$ has a non-negative solution. Let a solution be $\hat{c}_A$. We argue that $(c_A^*, c_B^*) = (\hat{c}_A, \psi(H(\hat{c}_A)))$ is an equilibrium. Indeed, if $(c_A^*, c_B^*) = (\hat{c}_A, \psi(H(\hat{c}_A)))$ then $\lambda_A^* = H(\hat{c}_A)$ and $\lambda_B^* = G(\psi(H(\hat{c}_A)))$. Since $\mu(\hat{c}_A) = 0$ then $\Pi_A(\hat{c}_A, \lambda_A^*, \lambda_B^*) = 0$. Similarly, $\Pi_B(c_B^*, \lambda_A^*) = 0$. Therefore, by construction, $\hat{c}_A$ is the activist's best response to $\psi(H(\hat{c}_A))$, and $\psi(H(\hat{c}_A))$ is the bidder's best response to $\hat{c}_A$. Therefore, $(c_A^*, c_B^*) = (\hat{c}_A, \psi(H(\hat{c}_A)))$ is an equilibrium, as required.

Next, we prove that if $c_A^* > 0$ then c_A^* is given by the solution of $\mu(x) = 0$, and if $c_A^* = 0$ then $\mu(0) \leq 0$. First, suppose on the contrary $c_A^* > 0$ and $\mu(c_A^*) \neq 0$. Notice that $\mu(c_A^*) =$

$\Pi_A(c_A^*, \lambda_A^*, \lambda_B^*)$, and by definition, the activist searches if and only if $c_A \leq c_A^*$. If $\mu(c_A^*) > 0$ ($\mu(c_A^*) < 0$) then from continuity there is $\varepsilon > 0$ such that if $c_A \in (c_A^*, c_A^* + \varepsilon)$ ($c_A \in (c_A^* - \varepsilon, c_A^*)$) then $\Pi_A(c_A, \lambda_A^*, \lambda_B^*) > 0$ ($\Pi_A(c_A, \lambda_A^*, \lambda_B^*) < 0$), thereby contradicting the optimality of c_A^*. Second, suppose on the contrary, $c_A^* = 0$ and $\mu(0) > 0$. Notice that $\mu(0) = \Pi_A(0, \lambda_A^*, \lambda_B^*)$. From continuity, there is $\varepsilon > 0$ such that if $c_A \in (0, \varepsilon)$ then $\Pi_A(c_A, \lambda_A^*, \lambda_B^*) > 0$, thereby contradicting the optimality of $c_A^* = 0$.

Last, we prove that there is $\overline{\gamma} > 0$ such that if $\gamma \in [0, \overline{\gamma}]$ then $c_A^* > 0$ in any equilibrium. Let $\mu(x, \gamma)$ be the function $\mu(x)$ parametrized by γ, and notice that $\mu(x, \gamma)$ is continuous in γ. Since $\mu(0, 0) > 0$ (whenever $b < \infty$), from continuity, there is $\overline{\gamma} > 0$ such that if $\gamma \in [0, \overline{\gamma}]$ then $\mu(0, \gamma) > 0$. Since we showed that an equilibrium always exists and if $c_A^* = 0$ then $\mu(0, \gamma) \leq 0$, it follows that if $\gamma \in [0, \overline{\gamma}]$ then in any equilibrium $c_A^* > 0$. ■

Remark: The next auxiliary lemma is used in the proof of Proposition 3 below. Its proof is given in Appendix C.

Lemma 7 *Suppose $\gamma \in [0, \overline{\gamma}]$ and θ^* is either $\underline{\theta}^*$ or $\overline{\theta}^*$. Let $\hat{\rho} \equiv \frac{\kappa/L - \gamma q}{s}$. Then:*

(i) If $b < \hat{\rho}$ then:

(a) c_A^* strictly increases in L, strictly decreases in γ, and does not change with κ.

(b) c_B^* does not change with respect to L, κ, or γ, and strictly decreases in b and s.

(ii) If $0 \leq \hat{\rho} < b$, then both c_A^* and c_B^* strictly increase in L and strictly decrease in κ.

(iii) If $\hat{\rho} < 0$, then both c_A^* and c_B^* strictly increase with L and do not change with κ.

Proof of Proposition 3. Recall that we defined $\hat{\rho} = \frac{\kappa/L - \gamma q}{s}$ in Lemma 7. Hence, by (3), $\rho(L) = \max\{0, \hat{\rho}\}$. Consider part (i) of the proposition, and note that $b < \rho(L)$ implies $\theta^* = G(c_B^*) \int_b^\infty dF(\Delta)$. Since $\int_b^\infty dF(\Delta)$ does not change with respect to L, κ, γ, or s, and decreases with b, the result follows from Lemma 7 part (i) and the observation that $\rho(L) = \hat{\rho}$. Consider part (ii) of the proposition, and note that $b \geq \rho(L)$ implies that

$$\theta^* = G(c_B^*) \left[\int_b^\infty dF(\Delta) + H(c_A^*) \int_{\rho(L)}^b dF(\Delta) \right]. \tag{20}$$

Since $\frac{\partial \hat{\rho}}{\partial \kappa} > 0$ and $\frac{\partial \hat{\rho}}{\partial L} < 0$, the result follows from Lemma 7 part (ii). If $\hat{\rho} < 0$ then based on Lemma 7 part (iii), θ^* increases with L and does not change with κ. ■

Proof of Proposition 4. We prove that $AR(\chi)$ is given by (12) case by case. First, if $\chi = 13D$ then prior to the announcement the activist bought L shares. Therefore, with probability $\frac{1}{2}$ there was no liquidity demand and the price was $p_i(L)$, and with probability $\frac{1}{2}$ there was a liquidity demand and the price was $p_i(2L)$. Either way, after the announcement the price jumps to $p_i(2L)$. Therefore, conditional on the 13D filing, the average abnormal return is

$$AR(13D) = p_i(2L) - \tfrac{p_i(2L)+p_i(L)}{2}. \tag{21}$$

Substituting $p_i(z_i)$ with the expressions in (5) gives the result. Second, if $\chi = takeover|13D$ then prior to the announcement on the takeover the price was $p_i(2L)$. Based on Proposition 1, if a merger is announced then $\min\{b, \rho(L)\} \leq \Delta$ and the share price converges to the takeover offer, which is given by $q + s\Delta + \mathbf{1}_{\{b\leq\Delta\}}(1-s)b$. Therefore, conditional on a 13D filing, the average abnormal return is

$$AR(takeover|13D) = \frac{\int_{\min\{b,\rho(L)\}}^{\infty}\left[q+s\Delta+\mathbf{1}_{\{b\leq\Delta\}}(1-s)b-p_i(2L)\right]dF(\Delta)}{\int_{\min\{b,\rho(L)\}}^{\infty}dF(\Delta)} = \left[\frac{1}{1-F(\min\{b,\rho(L)\})} - \lambda_B\right]v(L), \tag{22}$$

where we used (4) and substituted $p_i(2L)$ with the expression in (5). Third, if $\chi = takeover|\emptyset$ then prior to the announcement on the takeover the price was $p_i(0)$.[36] Based on Proposition 1, if a takeover is announced then $b \leq \Delta$ and the share price converges to the takeover offer, which is given by $q + s\Delta + (1-s)b$. Therefore, conditional on no 13D filing, the average abnormal return is

$$AR(takeover|\emptyset) = \frac{\int_{b}^{\infty}[q+s\Delta+(1-s)b-p_i(0)]dF(\Delta)}{\int_{b}dF(\Delta)} = \left[\frac{1}{1-F(b)} - \lambda_B\tfrac{1-\lambda_A}{N-\lambda_A}\right]v(0), \tag{23}$$

where we used (4) and substituted $p_i(0)$ with the expression in (5).

Last, we note that if $b < \rho$ then $AR(13D)$ does not change with L, κ, or γ. In this region, $v(0) = v(L)$, and hence, $AR(13D) = \frac{\lambda_B^*}{2}(1-1/N)v(0)$. Note that $v(0)$ is independent of these three parameters. Also note that according to Lemma 7, λ_B^* does not change with these parameters when $b < \rho$. This completes the argument. ■

[36] Note that we invoked the assumption that the market maker of firm i does not observe 13D filings in other firms. If this assumption is relaxed, the market maker would use the information about other firms as follows. If there was a 13D filing in firm $j \neq i$, then the activist has identified firm j as the target, and a takeover of firm i never takes place. The price of firm i is q. If there was no 13D filing in any other firm, it must imply that the activist did not search. In this case, the price of firm i prior to the announcement of the takeover is $p_i(0,0,\lambda_B^*)$ instead of $p_i(0,\lambda_A^*,\lambda_B^*)$. Therefore, $AR(takeover|\emptyset) = [\frac{1}{1-F(b)} - \frac{\lambda_B}{N}]v(0)$, and the observation that $AR(takeover|13D) < AR(takeover|\emptyset)$ does not change.

B Proofs of Section 4

B.1 Proofs of Section 4.1.1

Proof of Lemma 6. If the incumbent board retains control of the target in the second round of negotiations, then shareholder value is $q + \mathbf{1}_{\{b \leq \Delta\}} \cdot [s\Delta + (1-s)b]$. If the activist or the bidder obtains control of the target board, an agreement will be reached in the second round and the expected shareholder value is $q + s\Delta$. Therefore, if $b \leq \Delta$ then neither the bidder nor the activist can win a proxy fight, and hence, they will not initiate one. Suppose $\Delta < b$ and the first round of negotiations failed. Shareholders will support whoever runs a proxy fight, knowing that in both cases an agreement will be reached in the second round of negotiations and that the expected shareholder value will be $q + s\Delta$. Therefore, if one player is going to run a proxy fight, the other player does not have incentives to run a proxy fight, since by doing so he will obtain the same profit but will in addition incur the cost κ. Consider the case where the bidder runs a proxy fight. If the bidder runs a proxy fight then his expected payoff is $(1-s)\Delta - \kappa$. If neither the bidder nor the activist runs a proxy fight, then the firm will remain independent, and the bidder's profit will be zero. Therefore, the bidder will run a proxy fight if and only if $\frac{\kappa}{1-s} \leq \Delta$. This completes part (i). Consider the case where the activist runs a proxy fight. If the activist runs a proxy fight then her expected payoff is $\alpha(q + s\Delta) - \kappa$. If neither the bidder nor the activist runs a proxy fight, then the firm will remain independent, and the activist's profit will be $\alpha q(1-\gamma)$. Therefore, the activist will run a proxy fight if and only if $\rho(\alpha) \leq \Delta$. This completes part (ii). ■

Proposition 5 *Suppose* $\frac{\kappa}{1-s} < b$.[37] *If the bidder identifies firm i as a target and the activist owns α shares of that firm, then the unconditional shareholder value of firm i is* $q + \hat{v}(\alpha)$ *where*

$$\hat{v}(\alpha) = \int_b^{\infty} [s\Delta + (1-s)b]\, dF(\Delta) + \int_{\frac{\kappa}{1-s}}^{b} [s\Delta + s\kappa]\, dF(\Delta) + \int_{\min\left\{\frac{\kappa}{1-s}, \rho(\alpha)\right\}}^{\frac{\kappa}{1-s}} s\Delta dF(\Delta). \quad (24)$$

Proof. We start by proving that if $b \leq \Delta$ the bidder pays $q + s\Delta + (1-s)b$ and takes over the target after the first round of negotiations. Based on Lemma 6, if $b \leq \Delta$ then neither the bidder nor the activist will run a proxy fight. Therefore, both the bidder and the incumbent board

[37] According to Lemma 6, if $\frac{\kappa}{1-s} < b$ then more than one equilibrium of the subgame that follows the first round of negotiations may exist. We assume that whenever there is an equilibrium in which the bidder runs a proxy fight, this equilibrium is in play. This selection tilts the analysis against our result that the activist has any effect on the outcome of the takeover, and ensures that the equilibrium in play is the one that obtains the highest shareholder value in the subgame.

expect that in the second round of negotiations they will reach an agreement with an expected premium of $s\Delta + (1-s)\, b$. Therefore, the bidder will not agree to pay more than this amount and the incumbent board will not accept less than this amount. They will reach an agreement in the first round of negotiations, in which the bidder pays a premium of $s\Delta + (1-s)\, b$.

Next, we prove that if $\frac{\kappa}{1-s} \leq \Delta < b$ the bidder pays an expected price of $q + s\Delta + s\kappa$ and takes over the target in the first round of negotiations. Recall the assumption that if there is an equilibrium in the subgame that follows the first round of negotiations in which the bidder runs a proxy fight, then this equilibrium is in play. Based on Lemma 6, if the first round of negotiations fails, the bidder will run a proxy fight if and only if $\frac{\kappa}{1-s} \leq \Delta < b$. In this case, the bidder will run and win the proxy fight if the first round of negotiations fails. In the second round, the expected premium is $q + s\Delta$, and the bidder's expected profit is $\Delta\,(1-s) - \kappa > 0$. In the first round of negotiations, shareholders would reject any offer lower than $q + s\Delta$, and accept any offer higher than that amount. If the bidder is the proposer, he will offer $q + s\Delta$, and both the board and the shareholders will accept it. If the board is the proposer, he will offer $q + s\Delta + \kappa$, which leaves the bidder with a profit of $\Delta\,(1-s) - \kappa > 0$, and hence, the bidder will accept this deal. Overall, the expected takeover premium is $q + s\Delta + s\kappa$, as required.

Next, we prove that if $\rho\,(\alpha) \leq \Delta < \frac{\kappa}{1-s}$ and the activist owns α shares in the target, the bidder pays $q + s\Delta$ and takes over the target after the first round of negotiations. Based on Lemma 6, if the first round of negotiations fails and $\rho\,(\alpha) \leq \Delta < \frac{\kappa}{1-s}$ then the bidder will not run a proxy fight but the activist will. Therefore, both the bidder and the incumbent board expect that in the second round of negotiations the bidder will negotiate with the activist and they will reach an agreement with expected premium of $s\Delta$. Therefore, the bidder will not agree to pay more than this amount and the incumbent board will not accept anything less than this amount. They will reach an agreement in the first round of negotiations, in which the bidder pays a premium of $s\Delta$, as required.

Last, we show that in all other cases, the target remains independent under the incumbent board's control. In all other cases, $\Delta < \min\left\{\frac{\kappa}{1-s}, b\right\}$ and either $\Delta < \min\left\{\rho\,(\alpha)\,, b\right\}$ or the activist is not a shareholder of the target. Based on Lemma 6, neither the bidder nor the activist will run a proxy fight. Since $\Delta < b$, the target remains independent under the incumbent board's control, as required. The proof is completed by noting that (24) is average of these four cases. ■

Remark: The proofs for Lemmas 3, 4, and 5, and Proposition 2, continue to hold under the assumptions of Section 4.1.1 with the exceptions that $v\,(\alpha)$ is replaced by $\hat{v}\,(\alpha)$, $\Gamma\,(\alpha, \lambda_B)$ is

replaced by

$$\hat{\Gamma}(\alpha, \lambda_B) = \gamma q \left[1 - \lambda + \lambda \int_0^{\min\left\{\frac{\kappa}{1-s}, b, \rho(\alpha)\right\}} dF(\Delta)\right], \text{[38]} \tag{25}$$

and $w(\alpha)$ is replaced by

$$\hat{w}(\alpha) = \int_{\min\left\{\frac{\kappa}{1-s}, b, \rho(\alpha)\right\}}^{\infty} \Delta dF(\Delta). \tag{26}$$

B.2 Proofs of Section 4.2.2

Proposition 6 *Suppose the bidder identifies firm i as a target and the activist owns α shares of that firm. Then, the unconditional shareholder value of firm i is $q + v(\alpha, \zeta)$, where $v(\alpha, \zeta)$ is an increasing function of α given by*

$$\begin{aligned} v(\alpha, \zeta) &= \int_b^{\infty} \left[s\Delta + (1-s)\max\left\{\zeta\Delta \cdot \mathbf{1}_{\{\alpha>0\}}, b\right\}\right] dF(\Delta) \\ &\quad + \int_{\min\{b, \rho(\alpha,\zeta)\}}^{b} \left[s\Delta + (1-s)\zeta\Delta\right] dF(\Delta). \end{aligned} \tag{27}$$

where

$$\rho(\alpha, \zeta) \equiv \max\left\{0, \frac{\kappa/\alpha - \gamma q}{s + \zeta(1-s)}\right\}. \tag{28}$$

Proof. Suppose the first round of negotiations fails. If the activist is not a shareholder of the target, then the analysis is the same as in Section 3, where $\alpha = 0$. Suppose the activist owns $\alpha > 0$ shares in the target firm, and consider two cases. First, we argue that if $b \leq \Delta$ then the activist never runs a proxy fight and the incumbent board reaches an agreement in which the bidder pays a premium of $s\Delta + (1-s)\max\{\zeta\Delta, b\}$. To see why, notice that with the activist's presence, the incumbent learns about the action than can increase firm value by $\zeta\Delta$. Since $b \leq \Delta$, under the incumbent board control, an agreement in which the bidder pays a premium of $s\Delta + (1-s)\max\{\zeta\Delta, b\}$ is always reached. On the other hand, once the activist obtains control of the board, the standalone value of the firm increases by $\zeta\Delta$, and hence, the activist will accept a takeover offer if and only if the premium is greater than $-\gamma q + \zeta\Delta$. However, shareholders would reject any offer with a premium smaller than $\zeta\Delta$. Therefore, if an agreement between the activist and the bidder is reached, the expected takeover premium is $s\Delta + (1-s)\zeta\Delta$. Therefore, the activist has no incentive to run a proxy fight. Second, we argue that if $\Delta < b$ and $\rho(\alpha, \zeta) \leq \Delta$ then the activist runs a proxy fight and reaches an agreement in

[38] Note that the second line inside the maximum term in (6) is $-\Gamma(\alpha, \lambda_B)$ and hence is replaced by $-\hat{\Gamma}(\alpha, \lambda_B)$ as well.

which the bidder pays a premium of $s\Delta + (1-s)\zeta\Delta$, and if $\Delta < b$ and $\Delta < \rho(\alpha, \zeta)$ then the firm remains independent under the incumbent's control. To see why, note that $\Delta < b$ implies that the incumbent refuses the sell the firm or implement the activist's proposal. Therefore, firm value is q, and shareholders always elect the activist if she decides to run a proxy fight. The activist has incentives to run a proxy fight if and only if

$$\alpha[q + s\Delta + (1-s)\zeta\Delta] - \kappa > \alpha(1-\gamma)q \Leftrightarrow \rho(\alpha, \zeta) \leq \Delta,$$

as required. Consider the first round of negotiations. All parties involved anticipate the dynamic above if the first round fails. Therefore, if $b \leq \Delta$ then the bidder pays $q + s\Delta + (1-s)\max\left\{\zeta\Delta \cdot \mathbf{1}_{\{\alpha>0\}}, b\right\}$ and takes over the target after the first round of negotiations, and if $\rho(\alpha, \zeta) \leq \Delta < b$ then the bidder pays $q + s\Delta + (1-s)\zeta\Delta$ and takes over the target after the first round of negotiations. In all other cases, the target remains independent. Hence, for any $\zeta \in [0,1]$, $v(\alpha, \zeta)$ is given by (27), concluding the proof. ■

Proposition 7 *(i) In any equilibrium, the activist buys shares of firm i if and only if she searched and identified it as a target, in which case, the activist buys L shares.*

(ii) The bidder searches for a target if and only if $c_B < c_B^*(\lambda_A, \zeta, b)$, where c_B^* is the bidder's best response function and λ_A is the probability the activist searches and owns L shares in the target. The following holds:

(a) For all $\zeta \in (0,1)$ there is $\bar{b}(\zeta) \in (0, \infty)$ such that if $b > \bar{b}(\zeta)$ then $\frac{\partial c_B^*}{\partial \lambda_A} > 0$ and if $b < \bar{b}(\zeta)$ then $\frac{\partial c_B^*}{\partial \lambda_A} < 0$.

(b) If $\zeta = 1$ then $\frac{\partial c_B^*}{\partial \lambda_A} < 0$ for all $b < \infty$.

Proof. Similar to the arguments in Lemma 3, the activist buys shares of firm i if and only if she searches and identifies it as a target, in which case, the activist buys L shares. Let the probability the activist searches be λ_A. Then, for any $\zeta \in [0,1]$, the bidder's expected profit is given by $\Pi_B(c_B, \lambda_A, \zeta, b) = -c_B + \psi_L(\lambda_A; \zeta, b)$ where,

$$\begin{aligned}
\psi_L(\lambda_A; \zeta, b) &= \lambda_A(w(L,\zeta) - v(L,\zeta)) + (1-\lambda_A)(w(0,\zeta) - v(0,\zeta)) \\
&= (1-s)(1-\lambda_A)\int_b^\infty (\Delta - b)\, dF(\Delta) \\
&\quad + (1-s)\lambda_A \left(\begin{array}{c} \int_{b/\zeta}^\infty \Delta(1-\zeta)\, dF(\Delta) + \int_b^{b/\zeta} (\Delta - b)\, dF(\Delta) \\ + \int_{\min\{b, \rho(L,\zeta)\}}^b \Delta(1-\zeta)\, dF(\Delta) \end{array} \right),
\end{aligned}$$

where

$$w(\alpha,\zeta)=\int_{\min\{b,\rho(\alpha,\zeta)\}}^{\infty}\Delta dF(\Delta) \tag{29}$$

is the expected surplus from the takeover conditional on the bidder has identified the target and the activist owns α shares in the target. Therefore, if the bidder expects the activist to search with probability λ_A, the bidder searches if and only if $c_B<c_B^*=\psi_L(\lambda_A;\zeta,b)$. Note that

$$\frac{\partial\psi_L}{\partial\lambda_A}=(1-s)\left[\int_{\min\{b,\rho(L,\zeta)\}}^{b}\Delta(1-\zeta)\,dF(\Delta)-\zeta\int_{b/\zeta}^{\infty}(\Delta-b/\zeta)\,dF(\Delta)\right]. \tag{30}$$

Suppose $\zeta\in(0,1)$. Since the first term in $\frac{\partial\psi_L}{\partial\lambda_A}$ is (weakly) increasing in b and the second term is strictly decreasing in b, $\frac{\partial\psi_L}{\partial\lambda_A}$ is strictly increasing in b. In addition,

$$\lim_{b\to 0}\frac{\partial\psi_L}{\partial\lambda_A}=-(1-s)\,\zeta\int_0^{\infty}\Delta dF(\Delta)<0, \tag{31}$$

and

$$\lim_{b\to\infty}\frac{\partial\psi_L}{\partial\lambda_A}=(1-s)\int_{\rho(L,\zeta)}^{\infty}\Delta(1-\zeta)\,dF(\Delta)>0.$$

Since $\frac{\partial\psi_L}{\partial\lambda_A}$ is continuous with respect to b, by the intermediate value theorem, for all ζ there exists $\bar{b}\in(0,\infty)$ such that $\left.\frac{\partial\psi_L}{\partial\lambda_A}\right|_{b=\bar{b}}=0$. Since $\frac{\partial\psi_L}{\partial\lambda_A}$ is strictly increasing in b, this completes part (ii.a). If $\zeta=1$ then

$$\left.\frac{\partial\psi_L}{\partial\lambda_A}\right|_{\zeta=1}=-(1-s)\int_b^{\infty}(\Delta-b)dF(\Delta), \tag{32}$$

which completes part (ii.b). ■

Remark: In the next proposition, we show that the activist is more resilient than the bidder to the commitment problem in takeovers. For this purpose, we consider a variant of the setup in Section 4.2.2 in which the bidder never approaches the target with a takeover offer, but instead, the activist can make a takeover offer of her own. We maintain the assumption that the value of the target increases by $\zeta\Delta$ if the activist's proposal is implemented. The activist's proposal can be implemented anytime by the target board after the activist becomes a stakeholder of the target, including after the failure of the second round of negotiations as well as after the acquisition of the target by the activist. For simplicity, we assume $\gamma=0$ and that the bargaining protocol between the activist and the target board is exactly the same with that of between the bidder and the target board.

Proposition 8 (Capacity to acquire) *Suppose the first round of negotiations fails and the*

activist owns α shares in the target. Then, the activist runs a proxy fight if and only if $\kappa/\alpha \leq \zeta\Delta < (1-\alpha)b$. Whenever the activist runs a proxy fight, she wins.

Proof. We solve the game backward. If the second round of negotiations succeeded and the target is acquired by the activist, then the activist implements her proposal if it has not been implemented yet. Therefore, the post takeover target value is $q + \zeta\Delta$. If the second round of negotiations failed and the firm remains independent (that is, its ownership structure did not change), there are two cases. First, if the activist controls the target board then she implements her proposal if it has not yet been implemented, and the target value is $q + \zeta\Delta$. Second, if the incumbent board retains control then he implements the proposal if and only if $b \leq \zeta\Delta$, and hence, the target value is $q + \mathbf{1}_{\{b \leq \zeta\Delta\}} \cdot \zeta\Delta$.

Next, consider the second round of negotiations. There are two cases to consider. First, suppose either the activist controls the target board, or the incumbent board retains control and $b \leq \zeta\Delta$. The activist's proposal is implemented whether or not the bid fails. For this reason, the activist will not offer more than $q + \zeta\Delta$ per share. Moreover, target shareholders will not accept offers lower than $q + \zeta\Delta$, since they can always reject the bid and obtain a value of $q + \zeta\Delta$ once the proposal is implemented. Therefore, whether or not target is acquired, the activist's payoff is $\alpha(q + \zeta\Delta)$ and the shareholder value is $q + \zeta\Delta$. Second, suppose incumbent board retains control and $b > \zeta\Delta$. If the negotiations fail the proposal will not be implemented and the activist's payoff would be αq. If the activist acquires the firm, her payoff is $q + \zeta\Delta - (1-\alpha)\,\pi_2$, where π_2 is the offer made to target shareholders. Therefore, the activist is willing to offer up to $q + \frac{\zeta}{1-\alpha}\Delta$ per share. The incumbent board and the activist will reach an agreement if and only if $b \leq \frac{\zeta}{1-\alpha}\Delta$. If $b > \frac{\zeta}{1-\alpha}\Delta$ then the takeover fails and the shareholder value is q. If $\zeta\Delta < b \leq \frac{\zeta}{1-\alpha}\Delta$ then the incumbent and the activist reach an agreement in which $\pi_2 \geq q + b > q + \zeta\Delta$. Therefore, target shareholders approve any agreement reached by the activist and the incumbent, and firm is acquired by the activist. In this case, the expected shareholder value is $q + s\frac{\zeta}{1-\alpha}\Delta + (1-s)\,b$.

Next, consider the proxy fight stage. There are three cases to consider. First, if $b \leq \zeta\Delta$ then the activist's payoff is $\alpha(q + \zeta\Delta)$ whether or not she gets the control of the board. Therefore, she has no reason to run and incur the cost of a proxy fight. Second, if $\zeta\Delta < b \leq \frac{\zeta}{1-\alpha}\Delta$ then the activist always loses the proxy fight if she decided to start one. The reason is that shareholders know that if they elect the activist they will get $q + \zeta\Delta$ whereas if they reelect the incumbent, the activist will takeover the target and pay shareholders on average $q + s\frac{\zeta}{1-\alpha}\Delta + (1-s)\,b$, which is strictly higher. Anticipating her defeat, the activist never runs a proxy fight in this region. Third, if $b > \frac{\zeta}{1-\alpha}\Delta$ then the shareholder value is $q + \zeta\Delta$ if the activist gets the control

of the board, and q otherwise. Therefore, shareholders always elect the activist if she runs a proxy fight. The activist's payoff is $\alpha(q+\zeta\Delta)-\kappa$ if she runs and wins a proxy fight, and αq otherwise. Therefore the activist runs a proxy fight if and only if $\kappa/\alpha \leq \zeta\Delta$. Combining this condition with $b > \frac{\zeta}{1-\alpha}\Delta$ yields $\kappa/\alpha \leq \zeta\Delta < (1-\alpha)b$, completing the proof. ■

C Supplemental material

C.1 Supplemental material for Section 3

Proof of Lemma 7. By Proposition 2, an equilibrium always exists. Suppose $\gamma \in [0,\overline{\gamma}]$. Let $\mu(x)$ be defined by (10) and note that

$$\mu(x) = -x + L \times \max\left\{0,\ \begin{array}{c} \frac{1}{2}G\left(\psi\left(H\left(x\right)\right)\right)\left[v\left(L\right) - \frac{H(x)}{N}v\left(L\right) - \frac{1-H(x)}{N}v\left(0\right)\right] \\ -\gamma q\left[1 - G\left(\psi\left(H\left(x\right)\right)\right)\int_{\min\{b,\rho(L)\}}^{\infty} dF\left(\Delta\right)\right] \end{array}\right\}. \tag{33}$$

According to Proposition 2, if (c_A^*, c_B^*) is an equilibrium then $\gamma \in [0,\overline{\gamma}]$ implies $c_A^* > 0$, and hence, $\mu(c_A^*) = 0$ and $c_B^* = \psi\left(H\left(c_A^*\right)\right)$.

We first provide the regulatory conditions under which $\mu(x) = 0$ has the smallest and greatest solutions, which we denote by $\underline{c}_A^*$ and $\overline{c}_A^*$. If $b \leq \rho\left(L\right)$ then $v\left(L\right) = v\left(0\right)$ and $\mu(x)$ becomes

$$\mu(x) = -x + L \times \max\left\{0, \tfrac{1}{2}G\left(\psi\left(0\right)\right)v\left(0\right)\tfrac{N-1}{N} - \gamma q\left[1 - G\left(\psi\left(0\right)\right)\int_{b}^{\infty} dF\left(\Delta\right)\right]\right\}, \tag{34}$$

which is strictly decreasing in x, and hence, the equilibrium is unique. Suppose $b > \rho\left(L\right)$. We prove that $\underline{c}_A^*$ and $\overline{c}_A^*$ exist if N is sufficiently large. Let

$$M(x) \equiv L\left[\begin{array}{c} \frac{1}{2}G\left(\psi\left(H\left(x\right)\right)\right)\left[v\left(L\right) - \frac{H(x)}{N}v\left(L\right) - \frac{1-H(x)}{N}v\left(0\right)\right] \\ -\gamma q\left[1 - G\left(\psi\left(H\left(x\right)\right)\right)\int_{\min\{b,\rho(L)\}}^{\infty} dF\left(\Delta\right)\right] \end{array}\right], \tag{35}$$

and $\hat{\mu}(x) \equiv \max\left\{M(x), 0\right\}$. Note that $\hat{\mu}(x) \equiv \mu(x) + x$. It can be seen that $\hat{\mu}(x)$ is bounded. Therefore, there exists B such that $\hat{\mu}(x) \in [0, B]$ for all x. Note that

$$\tfrac{\partial \hat{\mu}(x)}{\partial x} = \mathbf{1}_{\{M(x)>0\}} \cdot \tfrac{\partial M(x)}{\partial x}. \tag{36}$$

Moreover,

$$\lim_{N\to\infty}\frac{\partial M(x)}{\partial x}=g\left(\psi\left(H\left(x\right)\right)\right)h\left(x\right)\left(1-s\right)L\left(\int_{\rho(L)}^{b}\Delta dF\left(\Delta\right)\right)\left(\tfrac{1}{2}v\left(L\right)+\gamma q\int_{\rho(L)}^{\infty}dF\left(\Delta\right)\right), \tag{37}$$

Therefore, there is $N_0\in(0,\infty)$ such that if $N>N_0$ then $\frac{\partial M(x)}{\partial x}>0$, and therefore, $\frac{\partial\hat{\mu}(x)}{\partial x}\geq 0$. By Tarski's Fixed Point Theorem, $\hat{\mu}(x)$ has the least and greatest fixed points $\underline{c}_A^*$ and $\overline{c}_A^*$ on $[0,B]$. Throughout the rest of the proof we assume that $\underline{c}_A^*$ and $\overline{c}_A^*$ exist.

We continue by proving that if $c_A^*\in\{\overline{c}_A^*,\underline{c}_A^*\}$ then $\frac{\partial\mu(x)}{\partial x}|_{x=c_A^*}\leq 0$. To see why, suppose on the contrary that $\frac{\partial\mu(x)}{\partial x}|_{x=\overline{c}_A^*}>0$. Then, there exists $x'>\overline{c}_A^*$ such that $\mu(x')>0$. Since $\lim_{x\to\infty}\mu(x)=-\infty$, there exists $x''>x'$ such that $\mu(x'')<0$. Hence, by the intermediate value theorem, there exists $x^*\in(x',x'')$ such that $\mu(x^*)=0$. But then, x^* is an equilibrium which is strictly greater than $\overline{c}_A^*$, contradicting the definition of $\overline{c}_A^*$ as the greatest equilibrium. The proof that $\frac{\partial\mu(x)}{\partial x}|_{x=\underline{c}_A^*}\leq 0$ is similar. The case $\frac{\partial\mu(x)}{\partial x}|_{x=c_A^*}=0$ is a knife-edge case, in which the function $\mu(\cdot)$ is tangent to the x-axis at the equilibrium point c_A^*. Since we focus on local comparative statics, when the equilibrium continues to exist upon a small change in the parameter, this case will be ignored.

Next, consider part (i). Since $b<\hat{\rho}$ then $v\left(L\right)=v\left(0\right)$ and

$$\begin{array}{l} c_B^*=(1-s)\int_b^{\infty}(\Delta-b)dF\left(\Delta\right) \\ c_A^*=L\frac{1}{2}G\left(c_B^*\right)\left(1-1/N\right)\int_b^{\infty}\left[s\Delta+(1-s)b\right]dF\left(\Delta\right)-L\gamma q\left(1-G\left(c_B^*\right)\int_b^{\infty}dF\left(\Delta\right)\right) \end{array} \tag{38}$$

which are unique. Part (i.b) follows directly from the functional form above. The effect of L,κ, and γ and on c_A^* also follow directly from the functional form above. The only ambiguity is with respect to b and s. Parameter s also affects c_B^*, but in the opposite direction it affects c_A^*. The sign of the effect depends on the curvature of G. Parameter b suffers from the same ambiguity, but unlike s, it also has an ambiguous direct affect on c_A^*. This completes part (i.a).

Consider parts (ii) and (iii). Applying the implicit function theorem on $\mu(c_A^*,y)=0$, where $\mu(c_A^*,y)$ is $\mu(c_A^*)$ parameterized by $y\in\{L,\kappa,s,b,\gamma\}$, yields

$$\frac{dc_A^*}{dy}=-\frac{\frac{\partial\mu(x,y)}{\partial y}|_{x=c_A^*}}{\frac{\partial\mu(x,y)}{\partial x}|_{x=c_A^*}}. \tag{39}$$

Recall, $\frac{\partial\mu(x,y)}{\partial x}|_{x=c_A^*}<0$ for $c_A^*\in\{\overline{c}_A^*,\underline{c}_A^*\}$. Therefore, $sign\left(\frac{dc_A^*}{dy}\right)=sign\left(\frac{\partial\mu(x,y)}{\partial y}|_{x=c_A^*}\right)$. Notice

that

$$\frac{\partial\mu(x,y)}{\partial\kappa}|_{x=c_A^*} = L\left[\begin{array}{c} \frac{1}{2}g\left(\psi\left(\lambda_A^*\right)\right)\frac{\partial\psi\left(\lambda_A^*\right)}{\partial\kappa}\left[v\left(L\right)-\frac{\lambda_A^*}{N}v\left(L\right)-\frac{1-\lambda_A^*}{N}v\left(0\right)\right] \\ -\frac{1}{2}G\left(\psi\left(\lambda_A^*\right)\right)\frac{\partial\rho(L)}{\partial\kappa}s\rho\left(L\right)f\left(\rho\left(L\right)\right)\left(1-\frac{\lambda_A^*}{N}\right) \\ +\gamma q\left(-\frac{\partial\rho(L)}{\partial\kappa}f\left(\rho\left(L\right)\right)G\left(\psi\left(\lambda_A^*\right)\right)+g\left(\psi\left(\lambda_A^*\right)\right)\frac{\partial\psi\left(\lambda_A^*\right)}{\partial\kappa}\int_{\rho(L)}^{\infty}dF\left(\Delta\right)\right) \end{array}\right] \tag{40}$$

Note that $\rho\left(L\right)=\max\left\{0,\hat{\rho}\right\}$. Since $c_A^*>0$, if $0\leq\hat{\rho}<b$ then $\frac{\partial\psi\left(\lambda_A^*\right)}{\partial\kappa}<0$. Also, based on (3), $\frac{\partial\rho(L)}{\partial\kappa}>0$. Therefore, $\frac{\partial\mu(x,y)}{\partial\kappa}|_{x=c_A^*}<0$. We conclude that $\frac{dc_A^*}{d\kappa}<0$. Notice that if $\hat{\rho}<0$ then $\frac{\partial\psi\left(\lambda_A^*\right)}{\partial\kappa}=0$ and $\frac{\partial\rho(L)}{\partial\kappa}=0$, and hence, $\frac{dc_A^*}{d\kappa}=0$. Similarly,

$$\frac{\partial\mu(x,y)}{\partial L}|_{x=c_A^*} = \frac{\mu(c_A^*,y)+c_A^*}{L}+L\left[\begin{array}{c} \frac{1}{2}g\left(\psi\left(\lambda_A^*\right)\right)\frac{\partial\psi\left(\lambda_A^*\right)}{\partial L}\left[v\left(L\right)-\frac{\lambda_A^*}{N}v\left(L\right)-\frac{1-\lambda_A^*}{N}v\left(0\right)\right] \\ -\frac{1}{2}G\left(\psi\left(\lambda_A^*\right)\right)\frac{\partial\rho(L)}{\partial L}s\rho\left(L\right)f\left(\rho\left(L\right)\right)\left(1-\frac{\lambda_A^*}{N}\right) \\ \gamma q\left(-\frac{\partial\rho(L)}{\partial L}f\left(\rho\left(L\right)\right)G\left(\psi\left(\lambda_A^*\right)\right)+g\left(\psi\left(\lambda_A^*\right)\right)\frac{\partial\psi\left(\lambda_A^*\right)}{\partial L}\int_{\rho(L)}^{\infty}dF\left(\Delta\right)\right) \end{array}\right] \tag{41}$$

Note that $\frac{\partial\psi\left(\lambda_A^*\right)}{\partial L}\geq 0$ and $\frac{\partial\rho(L)}{\partial L}\leq 0$. Moreover, $c_A^*>0$ since $\gamma\in[0,\overline{\gamma}]$, and $\mu(c_A^*,y)=0$ since c_A^* is an equilibrium. Therefore, $\frac{\partial\mu(x,y)}{\partial L}|_{x=c_A^*}>0$. We conclude that $\frac{dc_A^*}{dL}>0$.

Finally, to complete the proof, we note that $c_B^*=\psi(H\left(c_A^*\right))$. Therefore, $\frac{dc_B^*}{dy}=\frac{\partial\psi(\lambda_A^*)}{\partial y}+\frac{\partial\psi(\lambda_A^*)}{\partial c_A^*}\frac{dc_A^*}{dy}$. Since $\frac{dc_A^*}{dL}>0$, $\frac{\partial\psi\left(\lambda_A^*\right)}{\partial L}\geq 0$, and $\hat{\rho}<b$ implies that $\frac{\partial\psi(\lambda_A^*)}{\partial c_A^*}>0$, we conclude that $\frac{dc_B^*}{dL}>0$ if $\hat{\rho}<b$. Next, $0\leq\hat{\rho}<b$ implies that $\frac{\partial\psi\left(\lambda_A^*\right)}{\partial\kappa}<0$, $\frac{dc_A^*}{d\kappa}<0$, and $\frac{\partial\psi(\lambda_A^*)}{\partial c_A^*}>0$. Hence, we conclude that $\frac{dc_B^*}{d\kappa}>0$ if $0\leq\hat{\rho}<b$. On the other hand, $\hat{\rho}<0$ implies that $\frac{\partial\psi\left(\lambda_A^*\right)}{\partial\kappa}=0$ and $\frac{dc_A^*}{d\kappa}=0$. Hence, we conclude that $\frac{dc_B^*}{d\kappa}=0$ if $\hat{\rho}<0$. ■

Conditions under which multiple equilibria exist. The multiplicity of equilibria generated by strategic complementarity can be best seen when $N\to\infty$, $\gamma=0$, and $b\to\infty$. In this case, the target will sell only if the activist is present. Therefore, the buyer will find it optimal to search if and only if the activist is present, and the activist will search if and only if she believes that the buyer will be present. In particular, as a special case of Proposition 2, (c_A^*,c_B^*) must satisfy

$$\begin{array}{c} c_B^*=\left(1-s\right)H\left(c_A^*\right)\int_{\rho(L)}^{\infty}\Delta dF\left(\Delta\right) \\ c_A^*=s\frac{L}{2}G\left(c_B^*\right)\int_{\rho(L)}^{\infty}\Delta dF\left(\Delta\right) \end{array} \tag{42}$$

Notice that $(c_A^*,c_B^*)=(0,0)$ is always an equilibrium. That is, if the other player does not

search for sure, the transaction will never take place, and hence, there are no incentives to search. However, if

$$g(0)\,h(0) > \left(s(1-s)\,\tfrac{L}{2}\left[\int_{\rho(L)}^{\infty} \Delta dF(\Delta)\right]^2 \right)^{-1} \tag{43}$$

then there is a sufficiently large mass of buyers and activists with low search costs, and an equilibrium where $c_A^* > 0$ and $c_B^* > 0$ also exists. This result can be seen by noting that c_A^* must solve $c_A = s\frac{L}{2} G\left((1-s)\,H(c_A)\int_{\rho(L)}^{\infty}\Delta dF(\Delta)\right)\int_{\rho(L)}^{\infty}\Delta dF(\Delta)$. As $c_A \to \infty$, the RHS converges to a finite number and the LHS to infinity. The derivative of the RHS is $s(1-s)\,\frac{L}{2} g\left((1-s)\,H(c_A)\int_{\rho(L)}^{\infty}\Delta dF(\Delta)\right) h(c_A)\left[\int_{\rho(L)}^{\infty}\Delta dF(\Delta)\right]^2$. Therefore, if (43) holds, this derivative evaluated at $c_A = 0$ is strictly greater than one, and hence, an interior solution must exist. ■

Examples for Section 3.4.

We first provide the example of the ex-ante probability of takeover decreases with b, the incumbent's private benefits of control. We set the parameters as follows: $L = 0.10$, $N = 100$, $\gamma = 0$, $s = 0.95$, $\kappa = 0.03$, $F(\Delta) = 1 - e^{-\Delta}$, and $G(c) = H(c) = 1 - e^{-50c}$. In this example, the probability of takeover decreases from 91.79% to 47.85% as b increases from zero to 1.5, but then *increases* to 54.5%.

Next we provide an example where the ex-ante probability of takeover increases with γ, the activist's bias for early liquidation. We set the parameters as follows: $L = 0.05$, $N = 100$, $s = 0.9$, $\kappa = 0.06$, $b = 3$, $q = 3$, $F(\Delta) = 1 - e^{-\Delta}$ if $\Delta \geq 0$, and $G(c) = H(c) = 1 - e^{-100c}$. In this example, the probability of takeover increases from around 27% to 90% as γ increases from $\gamma = 0$ to $\gamma = 0.35$.

Finally, we provide an example where the average abnormal returns to announcement of 13D filing of the activist, $AR(13D)$, increase as with κ. We set the parameters as follows: $L = 0.05$, $N = 10$, $s = 0.75$, $\gamma = 0.2$, $b = 75$, $q = 100$, and chose lognormal distributions, specifically $F(\Delta)$ following a lognormal distribution whose corresponding normal has mean 3.85 and standard deviation 0.7, and $G(c) = H(c)$ following a lognormal distribution whose corresponding normal has mean 0.1 and standard deviation 0.01. In this example, $AR(13D)$ increases from 17.64% to 18.35% if κ increases from $\kappa = 0$ to $\kappa = 1.65$, but decreases for larger κ. ■

C.2 Rival's private benefits from control

In this section we analyze the baseline model under the assumption that the activist and the bidder can divert a non-trivial amount of corporate resources as private benefits after winning control of the target board, if the target remains independent. We denote the value transfer by $q\eta$, where $\eta \in (0,1)$. In the baseline model, $\eta \to 0$. For simplicity, we assume that the transfer involves no deadweight loss.

Lemma 8 *In the second round of negotiations, the target is acquired by the bidder unless the incumbent board retains control and $\Delta < b$, or the activist obtains control and $\Delta < \beta(\alpha, \eta)$ where*

$$\beta(\alpha, \eta) \equiv q[\eta(1-\alpha)/\alpha - \gamma(1-\eta)]. \tag{44}$$

Moreover, the expected shareholder value is given by

$$\Pi_{SH}(\Delta) = \begin{cases} q + \mathbf{1}_{\{b \leq \Delta\}} \cdot [s\Delta + (1-s)b] & \text{if the incumbent board retains control,} \\ (1-\eta)\, q & \text{if the bidder controls the board,} \\ (1-\eta)\, q & \text{if the activist controls the board and } \beta(\alpha,\eta) > \Delta, \\ q + s\Delta + (1-s)\, m(\alpha,\eta) & \text{if the activist controls the board and } \beta(\alpha,\eta) \leq \Delta. \end{cases} \tag{45}$$

where

$$m(\alpha, \eta) \equiv \max\{-\eta q, \beta(\alpha, \eta)\} \tag{46}$$

Proof. There are three scenarios to consider. In the first scenario, the incumbent board retains control of the target. Then the proof is identical to the first scenario of Lemma 1. In the second scenario, the bidder controls the board. With control, the bidder uses the board's authority to sign on a deal that offers target shareholders the lowest amount they would accept. Moreover, by controlling the target board, the bidder can extract ηq from the target's standalone value. Therefore, in this case, the bidder offers shareholders $(1-\eta)q$, and shareholders, who at this point cannot prevent the bidder from extracting ηq, accept this offer. In the third scenario, the activist controls the target board. If no agreement is reached with the bidder, the activist's payoff is $\alpha q(1-\eta)(1-\gamma) + q\eta$. Abusing her control of the board, the activist extracts $q\eta$ from the target. In addition, each share of the target has a value of $q(1-\eta)(1-\gamma)$, which is the standalone value of the target from the activist's perspective, taking into account the adverse effect of the value extraction and the activist's higher discount rate, $1-\gamma$. The activist agrees to sell the firm if and only if her proceeds from the takeover

are higher than $\alpha q\left(1-\eta\right)\left(1-\gamma\right)+q\eta$, which holds if and only if $q+\beta(\alpha,\eta)\leq\pi_2$. Once the activist has control of the board, shareholders would vote to approve the takeover if and only if the price is higher than $q(1-\eta)$, and the bidder will never offer more than $q+\Delta$. The bidder and the activist reach an acquisition agreement that is acceptable to shareholders if and only if $\beta(\alpha,\eta)\leq\Delta$. If $\beta(\alpha,\eta)>\Delta$ then the firm remains independent, and the long term shareholder value is $q\left(1-\eta\right)$. If $\beta(\alpha,\eta)\leq\Delta$ then the firm is sold to the bidder in the second round of negotiations. With probability s the activist offers $\pi_2=q+\Delta$, an offer which is always accepted by the bidder and the target shareholders, and with probability $1-s$ the bidder offers $\pi_2=q+\max\left\{-\eta q,\beta(\alpha,\eta)\right\}$, which is always accepted by the activist and the shareholders. ■

Lemma 9 *Suppose the first round of negotiations fails. Then:*

(*i*) *The bidder never runs a proxy fight.*

(*ii*) *If the activist owns α shares in the target, the activist runs a proxy fight if and only if*

$$\rho(\alpha,\eta)\leq\Delta<b\quad or\quad b+\frac{\kappa/\alpha}{1-s}\leq\beta(\alpha,\eta)\leq\Delta, \tag{47}$$

where

$$\rho(\alpha,\eta)\equiv\max\left\{\beta(\alpha,\eta),-\frac{1-s}{s}m(\alpha,\eta),\frac{\kappa/\alpha-q\gamma}{s}-\frac{1-s}{s}m(\alpha,\eta)\right\}. \tag{48}$$

Whenever the activist runs a proxy fight, she wins.[39]

Proof. Consider part (i). Based on Lemma 8, without the ability to commit not to abuse the power of the board, target shareholders are always worse off if they elect the bidder. Since $\kappa>0$, no party initiates a proxy fight she expects to lose. Hence, in equilibrium the bidder never runs a proxy fight. Consider part (ii). Based on Lemma 8, if $\beta(\alpha,\eta)>\Delta$ and the activist obtains control, shareholder value is $q\left(1-\eta\right)$, and therefore, shareholders would support the incumbent. Suppose $\beta(\alpha,\eta)\leq\Delta$. There are two cases to consider. First, if $b\leq\Delta$ then under the incumbent's control $\Pi_{SH}\left(\Delta\right)=q+s\Delta+(1-s)b$, while under the activist's control $\Pi_{SH}\left(\Delta\right)=q+s\Delta+\left(1-s\right)m(\alpha,\eta)$. Therefore, shareholder support the activist if and only if

[39] If $b+\frac{\kappa/L}{1-s}\leq\beta\left(L,\eta\right)\leq\Delta$ then shareholders do not elect the activist to the board because otherwise the incumbent would block the takeover (as in the baseline model), but rather, they elect the activist since she can negotiate a higher takeover premium, similar to our analysis in Section 4.2.1.

$b \leq m(\alpha, \eta)$. Since $b \geq 0$, this condition becomes $b \leq \beta(\alpha, \eta) \leq \Delta$. The activist runs a proxy fight only if

$$\alpha \left[q + s\Delta + (1-s)\,\beta(\alpha, \eta)\right] - \kappa \geq \alpha \left[q + s\Delta + (1-s)\,b\right] \Leftrightarrow \beta(\alpha, \eta) \geq b + \tfrac{\kappa/\alpha}{1-s}. \tag{49}$$

Combined, the activist runs a proxy fight if and only if $b + \frac{\kappa/\alpha}{1-s} \leq \beta(\alpha) \leq \Delta$, as required. Second, suppose $\Delta < b$. Under the incumbent's control $\Pi_{SH}(\Delta) = q$, while under the activist's control $\Pi_{SH}(\Delta) = q + s\Delta + (1-s)\,m(\alpha, \eta)$. Therefore, shareholders support the activist only if $-\frac{1-s}{s} m(\alpha, \eta) \leq \Delta$. Combined, the condition becomes

$$\max\left\{\beta(\alpha, \eta), -\tfrac{1-s}{s} m(\alpha, \eta)\right\} \leq \Delta < b. \tag{50}$$

Provided the activist is getting the support from shareholders, she runs a proxy fight if and only if

$$\alpha \left[q + s\Delta + (1-s)\,m(\alpha, \eta)\right] - \kappa \geq \alpha q\,(1-\gamma) \Leftrightarrow \tfrac{\kappa/\alpha - q\gamma}{s} - \tfrac{1-s}{s} m(\alpha, \eta) \leq \Delta \tag{51}$$

Combined, the activist initiates a proxy fight if and only if $\rho(\alpha, \eta) \leq \Delta < b$, as required. ■

Proposition 9 *Suppose the bidder identifies firm i as a target and the activist owns α shares of that firm. Then, the unconditional shareholder value of firm i is $q + \tilde{v}(\alpha)$, where $\tilde{v}(\cdot)$ is given by*

$$\begin{aligned} \tilde{v}(\alpha) \;=\; & \int_b^{\infty} [s\Delta + (1-s)b]\, dF(\Delta) + \int_{\min\{b, \rho(\alpha,\eta)\}}^{b} [s\Delta + (1-s)\,m(\alpha, \eta)]\, dF(\Delta) \\ & + \mathbf{1}_{\left\{b + \frac{\kappa/\alpha}{1-s} \leq \beta(\alpha,\eta)\right\}} \int_{\beta(\alpha,\eta)}^{\infty} (1-s)\,(\beta(\alpha, \eta) - b)\, dF(\Delta), \end{aligned} \tag{52}$$

Proof. Given Lemma 9 and Lemma 8, the proof is similar to the proof of Proposition 1, and for brevity, we only highlight the differences. Based on Lemma 9, if the activist owns α shares of the target and either $b + \frac{\kappa/\alpha}{1-s} \leq \beta(\alpha, \eta) \leq \Delta$ or $\rho(\alpha, \eta) \leq \Delta < b$, then the activist would run a successful proxy fight if the first round of negotiations fails. Based on Lemma 8, all players expect that once the activist obtains control of the board, she will reach a sale agreement in which the bidder pays in expectations $\pi_2'' = q + s\Delta + (1-s)\,m(\alpha, \eta)$ per share. Therefore, similar to the proof of Proposition 1, the incumbent and the bidder reach an agreement in the first round where the offer is π_2''. Note that if $b + \frac{\kappa/\alpha}{1-s} \leq \beta(\alpha, \eta) \leq \Delta$ then $0 < \beta(\alpha, \eta)$ and hence, $m(\alpha, \eta) = \beta(\alpha, \eta)$. In all other cases, if $b \leq \Delta$ the incumbent and the bidder reach an

agreement in the first round where the offer is $q+s\Delta+(1-s)b$, and if $b>\Delta$ the target remains independent under the incumbent board's control. This explains the term behind $\tilde{v}(\alpha)$. ■

Remark: In light of Proposition 9, the analysis of the search and position building phase does not change with the exception that when $\eta>0$ the term $v(\alpha)$ is everywhere replaced by $\tilde{v}(\alpha)$ and the term $w(\alpha)$ is everywhere replaced by $\tilde{w}(\alpha)=\int_{\min\{b,\rho(\alpha,\eta)\}}^{\infty}\Delta dF(\Delta)$.[40] Therefore, Lemmas 3, 4, and 5, as well as Proposition 2, continue to hold.

[40] Other exceptions are that the term $\Gamma(\alpha,\lambda_B)$ is replaced by $\tilde{\Gamma}(\alpha,\lambda_B)=\gamma q\left[1-\lambda\int_{\min\{b,\rho(\alpha,\eta)\}}^{\infty}dF(\Delta)\right]$, and the integral in the second line in (6) is replaced by $\int_{\min\{b,\rho(L,\eta)\}}^{\infty}dF(\Delta)$.