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How Shareholder Heterogeneity Distorts Corporate Voting

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Skewed Voting:

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When shareholders are heterogeneous, pessimistic shareholders can exit and sell their shares to more optimistic shareholders. This process results in a skewed distribution of valuations among current shareholders, which implies that the average share valuation is higher than the median. Thus, when shareholders collectively vote to sell the firm, a majority of shareholders (represented by the median) would accept underpriced and socially decreasing transactions (represented by the average).

I discuss the conditions, circumstances, and the normative implications arising from shareholders' skewed voting, such as the need for super-majority voting, granting the board of directors with a veto right in M&A deals ('poison pill'), and the purpose of the appraisal remedy. I re-examine these legal mechanisms and redescribe them as aimed to curb the tendency of the majority of shareholders to accept underpriced cash bids.

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1. Preface

Shares of a publicly traded company are held by a vast number of different shareholders with different expectations and heterogeneous valuations of the future value of their stock. There is ample empirical evidence indicating that shareholders are indeed heterogeneous and diverge in their valuations of their shares.

Shareholder heterogeneity, as Miller (1977) explains, causes pessimistic

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shareholders who have low valuations of their shares to exit the company and sell their shares to more optimistic and 'high-valuating' shareholders, a process that results in an increase in the share price. Similarly, I show that shareholder heterogeneity and the continual exit by pessimistic shareholders cause a *skew* in the shape of the distribution curve of current shareholders, there are more optimist than pessimist shareholders, and this skew in shareholders' valuations leads to several significant normative implications for corporate governance.

The skew in the distribution function can be explained by an extension to Miller's (1977) theory on shareholder heterogeneity: as pessimists sell their shares to more optimistic shareholders, there are expected to be more optimist than pessimistic shareholders. The skew to the right in the distribution function can also be explained by the fact that the distribution function is truncated and strictly bounded from the left by the market price (as no shareholders value their shares below market price) but not bounded from the right (as there is no upper limit to shareholders' high valuations). This asymmetry creates a one-sided right tail of optimistic shareholders that pushes the average to be higher than the median. A formal model is presented in the Appendix.

This observation, that the *average* share valuation by current shareholders is expected to be higher than the *median*, raises important normative consequences: a simple majority of shareholders represented by the median might vote to accept an inefficient and socially decreasing transfer of the company, as represented by the average. Hence, the simple-majority voting rule, which is currently the default rule, fails to protect heterogeneous shareholders from accepting underpriced and socially decreasing bids. I argue that the shareholders' voting processes should be adjusted to better represent shareholders' average preferences and not their median preferences.

I discuss several alternative corporate governance mechanisms that can be used to curb majority shareholders and protect shareholders from underpriced bids. The first-best solution is to raise the required voting threshold and implement a super-majority voting rule. A different, second-best, and more practical alternative is to subject any sale of the company to preliminary approval of the board of directors, a professional and sophisticated agent of the shareholders, who can negotiate with the bidder and demand a better price for all shareholders. The board's negotiated price would be higher than the price for which a simple majority of shareholders would have settled. I also

discuss several alternative mechanisms that could better align the corporate approval process with shareholders' average preferences.

This study explores the implications of shareholder heterogeneity on shareholders' collective decision making. It shows are shareholder heterogeneity causes a divergence between three different share valuations: the marginal share valuation represented by the market price, the median valuation, and the average valuation. I argue that if the objective of corporate law is to enhance shareholders' welfare, then its goal and point of reference should be to protect shareholders' average valuation, not the market price or the median valuation. The conclusions of this study are also relevant and shed valuable light on several highly controversial issues in corporate law, such as the question of directors' primacy, the debate over the veto power of the board of directors', the legitimacy of the 'poison pill', and shareholders' short-termism.

The conclusions of this research can also be generally extended to any other collective decision to sell a collectively held and heterogeneously valued asset in exchange for a homogeneous consideration (cash). When heterogeneous preferences are transferable, a skew is formed in the distribution of preferences by the group. Because of this skewed voting, some 'end-of-life' collective decisions must not be decided solely by a simple-majority vote, but instead must be supplemented (or replaced) by some other filtering mechanism that is better attuned to the group's average preference.

The rest of the paper is structured as follows: section 2 briefly reviews the background and relevant literature. Section 3 presents the basic model, and section 4 explores the normative implications for shareholders' voting and shareholders' compensation that arise from the divergence between marginal, median, and average share valuations. Section 5 describe the possible corporate governance mechanisms that can be used to address the problem of shareholders' skewed voting. Section 6 further discusses several corporate governance implications that arise from skewed voting, mainly in relation to hostile takeover bids. Section 7 discusses the major limitations of the model as to homogeneous consideration and generalizes the model to various circumstances of collective decision making. Section 8 extends the model to collective decisions to purchase a heterogeneously valued asset. Section 9 uses the model to analyze voting in controlled companies. Section 10 replies to several various possible

criticisms, and section 11 concludes.

2. Related Literature

There is ample empirical evidence supporting shareholder heterogeneity. One prominent line of research is the extensive research on the elasticity of the demand for stock, which shows that the demand for stock is not flat, as traditional finance theory argues, but in fact, curves downwards. Such research includes the addition or removal of shares from stock indexes, as explored in Shleifer (1986), Loderer et al. (1991), Kaul et al. (2000), Chakrabarti et al. (2002), and Levin & Wright (2006); Dutch auction repurchase plans (Bagwell 1991, 1992); returns around the expirations of IPO lock-up provisions (Ofek & Stern, 2000); the collapse of Internet stock (Schultz ,2008); and active money managers' fees (Petajisto, 2009). An additional line of research investigates blockholders' diversity and finds extensive differences between large shareholders, see Cronqvist & Fahlenbrach (2009), Edmans & Holderness (2017), and Schwartz-Ziv & Volkova (2020).

In the finance theory literature, Miller (1977) was the first to theorize that shareholder heterogeneity coupled with short-sale constraints will result in optimistic shareholding, and therefore an elevated stock price. [1] Miller's theory was later supported by several empirical researches, such as Chen et al. (2002) and Boehme et al. (2006).

Corporate law literature has paid very little attention to the issue of shareholder heterogeneity. Stout (1997, 2003) has been a prominent advocate against finance theory's traditional view of a homogeneous shareholding. Hayden and Bodie (2018) argue that a shareholders' franchise is based on the false premise of shareholders' homogeneity, and following the reality of shareholder heterogeneity, corporate theory should reconsider allowing other stakeholders to vote as well. More recently, Lipton (2018) argues that due to shareholder heterogeneity, corporate law should allow shareholders to 'divorce' the company through the appraisal remedy. However, none of them addressed the issue of corporate voting.

The research is also relevant for the basic flaw in the traditional voting systems,

[1] I further discuss Miller's (1977) theory and short-sale constraints in Sections 10.8 and 10.9.

which does not consider the intensity of voters' preferences. This flaw has led several scholars to suggest theoretical alternatives to the traditional corporate voting system. Levmore (2000) offers to permit vote buying. Posner & Weyl (2014) offers to use quadratic vote buying, and Listokin (2015) offers to adopt the Vickrey-Clarke-Groves voting mechanism in special settings, such as condominiums. The common denominator for these proposed voting mechanisms is that they demand shareholders to bear monetary costs in exchange for exercising the right to vote, which raises serious normative doubts and practical difficulties.

Lastly, very few works discuss the differences between median and average valuations of shares, and the legal consequences arising from these differences. Listokin (2008, 2009) finds evidence that the pivotal median shareholder in proxy contests exhibits different preferences compared to the 'price-setting' marginal shareholder. However, Listokin stipulates that this difference is attributed to management's ability to influence the voting process. Bernhardt, Liu, and Marquez (2018) study how shareholder heterogeneity affects acquiring a firm's management choice between offering the target shareholders cash or stock. They make a similar argument that distinguish between the marginal shareholder, who determines the target's market price, and the median shareholder, who determines the target's bid price. However, Bernhardt et al. (2018)'s work is mainly limited to the buyer's dilemma of cash versus stock consideration. Choi & Talley (2018) also note that the merger price depends on the legal threshold for target shareholders' approval vote. However, their focus is on appraisal remedy litigation, and they do not consider shareholder heterogeneity effects on corporate voting.

3. Theory of Skewed Voting

In this section, I wish to show how shareholder heterogeneity leads to a skewed distribution of valuations in which the average valuation is higher than the median. Later, in Sections 4- 6, I discuss the normative implications of this skew.

3.1 Intuitive Explanations

Before presenting a more formal model, I wish to offer an intuitive explanation of why

corporate voting is skewed and why the average shareholder valuation is higher than the median.

Consider the proposition that many if not most shareholders value their shares at or slightly more than the current market price. Fewer shareholders are more optimistic than the market and hold very high valuations for their shares. If the distribution of valuations among the shareholders is so, then such a distribution will be skewed to the right, and the highly optimistic shareholders will push the average share value to be above the median. Consider, also, that the distribution of shareholders' valuations is strictly bounded from the left by the market price (as no current shareholder values his shares below market price), but it is not bounded from the right (as there is no upper limit to optimistic shareholders' valuations). This causes an asymmetric distribution with a one-sided right tail, that pushes the average to be higher than the median.

3.2 The Model

Assume a heterogeneous group of market investors. The investors are heterogeneous in the sense that they attribute different values to the shares of a firm[2], and I assume these heterogeneous valuations are normally distributed.[3]

In t_1, shares of a firm are offered to the investors in exchange for a certain fixed market price (I ignore the prior shareholders of the firm). Each investor will then evaluate the value of the firm and its shares and, based on his private valuation, will decide whether to accept the offer and become a shareholder or decline. I assume that the firm would issue a share for every investor who accepted the offer and paid the price. For simplicity, I also assume that each investor can buy only up to one share (note that some limitation on the hoarding of shares is needed to maintain shareholder heterogeneity[4]).

In t_2 we observe the results. Any optimist investor who values the share above the

[2] I further discuss and define heterogeneity in Sections 10.1 and 10.2.
[3] This assumption of a normal distribution of valuations can be eased to any generally symmetric and convex function.
[4] Without some limitation or costs on the hoarding of stock, the one shareholder who holds the highest share valuation would simply buy all available stock in the market. The basic assumptions of stock market and shareholders' heterogeneity must also assume some practical limitations and costs of hoarding of stock. This subject is further discussed in section 10.2 below.

asking price has accepted the offer and become a shareholder. None of the pessimistic investors who value the shares below market price has purchased any shares. The distribution of valuations by the investors who became shareholders would be shaped like a normal function that was cut (truncated) from the left by the market price and has a tail to the right created by the more optimistic shareholders. The distribution curve would be *asymmetric* and skewed to the right, which means that the average share valuation would be higher than the median. A formal proof to this claim in attached below in the Appendix.

Note, that the shape of the distribution curve is expected to remain skewed to the right, even after we allow for trade in the secondary market. Pessimistic investors who value the shares below market price will continue to refuse to buy any shares and thus will remain out of the firm. Optimist shareholders who value the shares above the market price will continue to hold only one share due to the costs of hoarding. Note also that the model does not depend on the price for the shares, and whether the market price is high or low will not change the general skewed shape of the distribution curve.

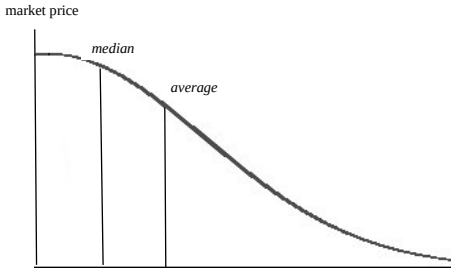
The distribution of shareholders' valuations is expected to remain skewed even if we relax a bit the limitation regarding the hoarding of shares. As the formal model shows, as long as the distribution is generally normal (symmetric) and truncated from the left by the market price, we can expect the average share valuation to be higher than the median.

4. Normative Implications

Our finding above of a skew in shareholders' valuations and of the average valuation being higher than the median implies several normative conclusions.

4.1 Marginal, Median, and Average Valuations

Let us start with an observation of the divergence between the marginal, median, and the average shareholder's valuation, and the unique implication of each of them in corporate law:



Distribution of Valuations in a Public Company

(i) *The marginal (market) valuation* – is held by the marginal shareholder who currently trades the shares in the market, and hence the marginal shareholder determines the current market price. In many common legal circumstances, the market value of the marginal shares determines the 'fair' value of *all* shares in the company.

(ii) *The median valuation* – is held by the median shareholder and determines the outcome of a simple majority vote by the shareholders. A bid to buy all the shares of the company will obtain the approval of a simple majority of shareholders if the proposed bid price is set at or above the median value.

(iii) *The average valuation* - is the average value of all shares of the company held by its heterogeneous shareholders. If we define efficiency by the Kaldor-Hicks criterion, then the average share valuation determines the aggregated value of the company to its shareholders. A sale of the company and the transfer of its shares (merger) would be welfare-enhancing if the bid price per share is higher than the average share value.

I now turn to review the major two corporate governance implications resulting from this divergence between the marginal, median, and average valuations.

4.2 Shareholders' Voting

Voting is the basic mechanism for a corporation's collective decision-making, and the simple-majority voting rule is the most common and prevalent default voting rule.[5] But is the common default simple-majority voting rule successful in sorting between socially enhancing and socially decreasing collective transactions?

[5] See, for e.g., Sections §216 and §251 of the Delaware General Corporation Law (DGCL).

Assume a buyer (a bidder) proposes to buy a publicly held company from the hands of its shareholders for cash. For such a transfer to be welfare-enhancing, the buyer must offer to pay a total price that is higher than the aggregate value of all shares held by its shareholders.[6] The buyer's bid price per share must be higher than the *average* value of the shares; otherwise the transaction would be socially decreasing.

When shareholders are asked to approve or reject such an offer, they vote (in a merger) or give notice (in a tender offer) whether to accept or reject it, in accordance with their individual valuation. If an offer receives the approval of a simple majority of the shareholders, then the offer is deemed accepted and the collective decision is then enforced upon all shareholders who must also sell their stock to the bidder.[7] The critical point here is that the collective decision to sell is decided not by the *average* voter, but by the *median* voter and according to the median valuation of the company.

Only if we assume that the company has a *symmetric* distribution of valuations, only then would the median and the average coincide, and the simple majority rule would then produce an accurate voting outcome. However, in a publicly held firm, we must assume the opposite: that shareholders' valuations are *asymmetric* and skewed to the right. Therefore, the median valuation is systematically lower than the average, and the majority-voting rule is exposed to false-positive outcomes when shareholders approve to sell the company for a price below its aggregated value.

For the voting mechanism to successfully sort and distinguish between efficient and inefficient transfers, the threshold for shareholders' voting should be higher than

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[6] Note, that in this analysis I omit the bidder's own private valuation of the company and his potential gains from the transfer. It is possible that a bidder's per-share valuation is higher than the average shareholder, but the bidder will set the bid price below the average, to maximize his gains. In such a case, although shareholders would accept a transfer that would cause them an aggregate loss, such a transfer would still be socially desirable due to bidders' gains. However, to guarantee that the bidder's valuation does indeed exceed shareholders' average, there is alternative but to enable shareholders to set their reservation price at said average. Second, recall that corporate governance and shareholders' voting rules are designed by and for the benefit of shareholders. When shareholders design their voting mechanism, ex-ante, they may want to guarantee that no underpriced less-than-average transaction will take place, and they would find very little comfort in knowing that the bidder would gain from their losses. Third, if we assume that bidders operate in a competitive environment, then target companies should be able to extract most if not all of the bidders' gains. The empirical data supports this assertion, and even show that bidders tend to bid too high and often suffer from a 'winner's curse'. See discussion in Section 8.

[7] In both mergers and in tender offers, the simple majority decision can mandate a coerced sale of all shares of the company, see Section §251(c) and §251(h) of the DGCL.

the median and closer to the average voter. Therefore, I argue, that in a publicly traded firm, several 'end-of-life' decisions, such as the decision to sell, merge, or the dissolving of the company for cash, a *super-majority* voting rule is required.

Alternatively, such collective decisions cannot be left entirely to the hands of a simple majority of shareholders, and therefore must also be subject to various additional legal requirements, such as the approval of a Board of Directors. I discuss the various second-best alternative solutions in section 5 below.

4.3 Shareholders' Compensation

Shareholder heterogeneity and the divergence between marginal, median, and average valuations are also relevant to the question of shareholders' compensation.

Let us examine a situation where shareholders are subject to a coerced sale of the company, and they are unwillingly separated from their shares, for example, in an expropriation or a freeze-out. Usually in these situations, the court is asked by the former shareholders to grant them with an adequate monetary compensation, an Appraisal Right. The questions then arise, to what extent should the courts compensate shareholders, and which valuation of the shares should the court use?

For the expropriation of shares to be socially efficient, courts should compensate shareholders according to the *average* value of the expropriated shares. Otherwise, if shareholders were to receive a less-than-average value for their shares, then aggregately such an expropriation might be welfare-decreasing. However, courts tend to compensate shareholders based on the currently traded market price or a recent deal price as a fair assessment of their value. However, such a market-based compensation only compensates some marginal shareholders who valued their shares at market price and therefore hold the *lowest* possible valuation for the company. All other heterogeneous shareholders will be under-compensated. More importantly, from an aggregated social perspective, such an expropriation might be inefficient and socially decreasing.

This conclusion in not limited only to appraisal litigation, and it can be extended to any civil litigation where a plaintiff is entitled to a monetary compensation based on the value of her shares. Courts should reconsider the traditional use of the traded market

price as a conclusive evidence for the fair value of the shares. As explained, the market price only represents the lower boundary value of the shares. Courts should therefore allow plaintiffs to argue and bring evidence of higher values such as the average value.

5. Possible Solutions to Shareholders' Skewed Voting

Heterogeneous shareholders, faced with the risk that a majority of them might approve an underpriced and socially decreasing sale of the company, have mainly four alternative courses of actions: (i) raise the voting threshold to a super-majority voting rule; (ii) compensate the dissenting optimist shareholders; (iii) disallow cash as an acceptable consideration; (iv) subject any sale of the company to additional legal requirements, such as the approval of the board of directors. Lastly, I also discuss tenure voting.

5.1 Super-majority Voting

The first available solution to correct shareholders' voting is to adjust the voting threshold and raise it from the common simple-majority rule (of 50% needed) to a stricter voting rule that require a higher percentage of shareholders' approval.[8]

Such a solution has indeed been adopted in practice. Some states in the U.S. require a super-majority voting in some M&A deals. Such an example is Section 203§ of the General Delaware Corporate Law (DGCL), which states that in certain circumstances, a merger must be approved by a super-majority vote of 85% of shareholders. Super-majority voting rules have also been adopted by many companies themselves in their bylaws. As of 2018, 17% of all Russel's 3000 companies require a super-majority vote of over 55% of their shareholders to approve a merger (see ISS Analytics, 2019).

Currently, the adoption of super-majority voting rules is perceived by many as intended to serve as a defensive mechanism against 'hostile' takeovers (Ruback, 1987),

[8] A slightly different mechanism that achieves a similar goal is to count the shares that refrain from voting (or voted to abstain) as a "no" vote. This can be done be requiring a *majority voting*, meaning an active approval and a "yes" vote from a majority of all issued stock, as opposed to a more common *plurality voting*, which requires only that the number of "yes" votes would surpass the number of "no" votes.

and to allow the incumbent management to hold on to their positions, despite shareholders' discontent (Bebchuk et al. 2009).[9] However, as I show here, super-majority voting rules have a substantial social advantage and can enhance shareholders' welfare. Super-majority voting improves shareholders' voting process by better reflecting shareholders' aggregated preferences. Lawmakers and corporate founders should therefore consider more favorably adopting laws and bylaws that will elevate the voting threshold for mergers and acquisitions, and institutional advisors such as ISS should reconsider their current reservation from such proposals to adopt super-majority voting rules (ISS, 2019).

5.2 Compensation and the Appraisal Remedy

A second possible solution for the risk that majority of shareholders might approve an underpriced bid, is to offer some compensation to the dissenting optimistic 'high-valuating' shareholders. Such a mechanism is the appraisal remedy, which grants dissenting shareholders in a merger with the right to sue the company for the 'fair' value of their shares.[10] The prevailing rational is that the appraisal right is intended to compensate the dissenting shareholders for the forced taking of their shares. This research, however, offers an alternative rationale: the appraisal remedy raises the overall consideration paid by the bidder, and thus can block underpriced bids that otherwise would have been accepted by a simple majority of shareholders.

However, if the appraisal right should be used to better filter proposed mergers, and to guarantee that only welfare-enhancing transactions would take place, then current appraisal remedy laws should be modified and readjusted to fulfill such a task. One adjustment that lawmakers should consider is to allow for *all* shareholders of the target to receive appraisal compensation, and such compensation should be equal to the difference between the deal price and shareholders' average valuation. However, because only dissenting shareholders are currently entitled to sue for appraisal remedy, then an efficient appraisal remedy should grant dissenting shareholders with at least *twice* the difference between the deal price and average value. This is a reasonable default, as dissenting shareholders will always be no more than 50% of all shareholders

[9] The issue of hostile takeovers is further discussed in Sections 5.4 and 6.1 below.

[10] see Section §262 to the DGCL.

of the company.

5.3 Payment in Traded Stock and the Market-Out Exception

Another possible solution to the skew in the shareholders' voting process is to offer the shareholders traded stock instead of cash. This conclusion may be counterintuitive, as it contradicts the prevailing opinion that an all-cash bid is ‘safer’ and less coercive to shareholders than a stock-for-stock bid.[11]

As I show in Section 7.1 below, the problem of skewed voting and that a majority of shareholders would accept an underpriced bid, only occurs when the bidder offers the shareholders cash, or another type of a *homogenous* consideration. If the bidder offers shareholders a heterogeneous consideration, such as newly issued stock or other traded security, then shareholders would vote in a different manner and more likely would reject an underpriced bid. This distinction is further discussed in Section 7.1 below. For now, it is sufficient to say that in an all-cash bid, we can anticipate that shareholders' median valuation is lower than the average, and therefore we fear that the majority would accept an underpriced bid. However, in a stock-for-stock bid, the relationship between the median and the average are not so clear. Therefore, the risk of skewed voting risk is much lower in a stock-for-stock bid, and the current default of a simple-majority voting rule seems more suitable.

An interesting application of this distinction between cash and stock bids can be found in the market-out exception to the appraisal right. Generally, the appraisal right is denied from shareholders when the merger consideration is in traded stock, meaning once shareholders are given a 'market out'. A traditional explanation for the market-out exception is that in a stock-for-stock merger, shareholders are not completely forced-out of the company and are not totally deprived of the economic promise entailed in the stock. In a stock-for-stock merger, shareholders manage to maintain some affinity and an economic linkage to the target's business enterprise. By obtaining newly issued stock in the merged company, shareholders in a stock-for-stock merger are less in need for an appraisal remedy.

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[11] See, for example, the discussions held in the Delaware's court concerning 'hostile' takeovers and the legitimacy of the 'poison pill', cited in footnote 12 below.

This research offers another possible explanation to the market-out exception: In a cash-for-stock merger, the majority shareholders might approve an underpriced and socially decreasing merger, and therefore the appraisal remedy is needed to raise the merger price and thus block some 'bad' mergers. However, in a stock-for-stock merger, there is less of a risk that the majority will approve underpriced mergers; therefore, an appraisal right is less needed.

5.4 Additional Requirement - Board's Veto

A different kind of approach to confront the threat of skewed voting and of an underpriced bid accepted by the majority of shareholders is to subject any sale of the company to a different legal requirement, instead or in addition to the approval by the majority shareholders. Such an additional requirement can be the authorization of a professional regulator. Another common legal requirement is to subject the deal to the approval of the court, as done in Scheme of Arrangements.

A more common and conventional solution to the skewed voting problem is to require the additional approval of a professional agent acting on behalf of the shareholders, i.e., the Board of Directors. In the context of mergers, the requirement for the approval of the Board of Directors is set in U.S. states' laws, such is Section §251 of the DGCL, which requires the approval of the Board of Directors for any merger. It is interesting to note that even the DGCL views the board's approval and the super-majority requirement as exchangeable substitutes: Section §203 of the DGCL, which requires a supporting vote of 85% of shareholders, but this super-majority requirement can be waived by a decision of the board.

The legal requirement for board's approval is not limited to mergers, but also has been extended to the context of tender offers. Various courts' rulings[12] have essentially implemented such a requirement for boards' approval, indirectly, by permitting incumbent directors to thwart hostile takeovers using a 'shareholders rights plan', infamously known as the 'Poison Pill' (Davidoff & Thomas, 2016). Poison Pills

[12] For the evolutionary Delaware's case law permitting the use of 'poison pills' see: Unocal Corp. v. Mesa Petroleum Co., 493 A.2d 946, 954 (Del. 1985); Moran v. Household Int'l, Inc., 500 A.2d 1346, 1357 (Del. 1985); City Capital Associates Ltd. Partnership v. Interco Inc. 551 A.2d 787 (Del.Ch. 1988); Paramount Communications, Inc v Time, Inc 571 A2d 1140 (Del 1989); Unitrin, Inc. v. Am. Gen. Corp., 651 A.2d 1361, 1373 (Del. 1995); Air Prods. & Chems., Inc. v. Airgas, Inc., 16 A.3d 48, 105 (Del. Ch. 2011).

are, in essence, equivalent to granting the Board of Directors with a veto right to 'just say no' to any unwanted or underpriced takeover (Kahan & Rock, 2002; Bebchuk, 2002). I further discuss this issue in Section 6.1 below.

6. Discussion

6.1 Substantive Coercion and the 'Poison Pill'

The model above exposes a vulnerability for publicly held target companies: a sophisticated bidder can purchase a firm for a price below its worth to its shareholders. The bidder can propose shareholders to sell their shares for an asking price above the median but below the average valuation. Because the bid would be above the median, the majority of shareholders would accept it, and because the bid would be lower than the average, the total consideration paid to shareholders would be less than the aggregated value of the firm. Such a transfer would be inefficient and would decrease social welfare.

This scenario is very relevant to hostile takeovers and to the much long debate over, and the legitimacy of, the 'poison pill': A bidder may come along and issue a 'hostile' tender offer directly to the shareholders. The incumbent management, acting either out of self-interest to hold on to their lucrative positions in the company or acting altruistically to protect the interests of the shareholders from an underpriced bid, will then deploy a 'poison pill' that will thwart the hostile bid and deprive the shareholders of a premium bid. Management will commonly justify its actions by arguing that the proposed bid was too low and therefore detrimental to shareholders. On the other hand, by doing so, management deprives shareholders of the ability to decide for themselves whether to accept or reject the bid.

The argument that management can block a takeover bid solely on the basis that management's views the offered price as too low for shareholders to accept, has been figuratively named in the literature as “substantive coercion” (Gilson & Kraakman, 1988) and was accepted by the Delaware courts as a legitimate justification for the incumbent management to deploy a ‘poison pill’, which prevents shareholders from accepting such unauthorized bids. Delaware courts and many practitioners have

traditionally viewed management as the protector of the company's fortress, and as such, management can act to prevent naïve shareholders from letting the hostile 'barbarian' bidders from entering the company's gates. On the other hand, many critics view management's actions to block premium bids as motivated by management's self-interest to retain their lucrative positions in the company, and that the decision whether to accept or reject a hostile bid should be left entirely to the hands of its recipients, the shareholders, especially when the decision is based solely on the bid's price.

This debate over the role of boards of directors in hostile takeovers has been continuing for decades. See, for example, Bebchuk (2002) and Lipton (2002). This research suggests a novel argument for granting boards with an active rule and a veto right in takeovers, which are subject to a mere simple majority vote. As we have seen, shareholders' majority approval is not a sufficient condition to guarantee the efficiency of a transfer, and shareholders could benefit from subjecting a sale of the company to a veto right by an agent acting on their behalf.

6.2 Short-Termism

The research's conclusions are also applicable to the ongoing debate on shareholders' short-termism. In brief, there is a common perception that too many shareholders are concentrated in only short-term gains, and shareholders' pressure for short-term gains is detrimental to the 'true' corporate goal of pursuing long-term profit. Therefore, some practitioners have argued that boards should be isolated from shareholders' pressure to short-term gains and allowed to operate and govern independently of shareholders' preferences. On the other side, other critics have argued that there is a lack of evidence supporting short-termism by shareholders. Such a distinction is theoretically questionable, and isolating boards from shareholders' supervision is highly problematic. See Bebchuk (2013), Strine (2014).

This research does not aim to delve into the short-termism debate. My only aim is to point out that this model's conclusions may also be used to support the short-termism argument for isolating the boards: One can argue that there is a strong connection and a considerable overlap between pessimistic 'low-valuating' shareholders and short-term shareholders, and between optimistic and long-term shareholders. As I have shown, most shareholders are pessimists and are inclined to over-accept

underpriced bids, hence suffer from short-termism. Therefore, there is some merit in the argument for isolating boards from the preferences of shareholders.

6.3 Tenure Voting

Could a tenure voting mechanism be used as a solution for shareholders' skewed preferences? In tenure voting mechanism, shareholders that hold on to their shares for a long period of time (e.g., for three years) will receive more voting power (e.g., three votes per share) (Berger et al. 2016). In practice, tenure voting has been used by companies primarily as a substitute for dual-class capital structure, aimed to maintain the control of minority insiders (Berger et al. 2016). However, could tenure voting be used to strengthen the long-term preferences of shareholders?

In my opinion, as shareholders' valuations are not necessarily aligned with shareholders' past holding periods, tenure voting can offer only a limited solution to the problem of shareholders' skewed preferences and will perform poorly as a mechanism to distinct between efficient and inefficient bids.

6.4 Empirical Evidence

There is some evidence that supports the hypothesis that shareholders are inclined to overly approve mergers and acquisitions. In a recent study by Cox, Mondino & Thomas (2019), they show that from a large database of 1620 M&A deals from 1996 until 2017, only five deals - a mere 0.3% - were struck down by shareholders. Only 17 more deals (1%) were withdrawn before completion, and it is possible that some of them were withdrawn because of an anticipated negative shareholders' vote. Previous research has also found similar results (Cox at al., *id*, at footnote 20). The remarkably low rejection rates of shareholders votes in M&A deals have led Cox et al. (2019) to wonder whether shareholders' voting is of value at all.

There is very slim empirical evidence on shareholders' super-majority voting. Boone et al. (2017) studied a recent Delaware's amendment in Section 251(h) to the DGCL, which basically lowered the holding threshold needed for a two-tiered tender offer, from a 90% super-majority to a 50% simple-majority of shareholders. Boone et al. (2017) found that this amendment was welfare-enhancing for both bidders and target

companies. However, there are more than a few reasons why these findings may not be applicable to this project: First, their research deals with the unique circumstances of a two-tiered tender offer.[13] Second and more broadly, this kind of an empirical research may have explored the wrong questions. Clearly, the simple-majority voting rule is preferable to *bidders* - as it lowers the target's price for the bidder. The simple-majority rule is also beneficial to the *target company*'s market price – as the firm's market price tends to rise after a successful transaction, from the marginal value to the median value.

However, empirical research will struggle to find evidence whether the simple-majority rule is preferable to *shareholders* and whether a transaction is desirable from a social welfare perspective. As explained above, the mere fact that the market price of a target has risen, does not indicate that the shareholders are better off. Some transactions may be beneficial to shareholders while others may not. I will also discuss the challenge of future empirical research in the concluding remarks.

7. Limitations and Generalization

7.1 Limitation: Heterogeneous Consideration

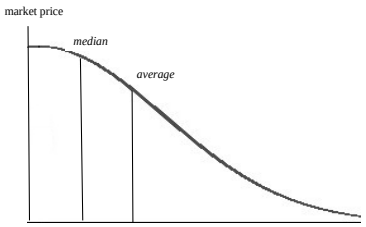
The model above and the analyses above are limited to cases where shareholders are offered *homogeneous* consideration, such as cash, in exchange for their shares. Otherwise, when shareholders are offered a *heterogeneous* consideration, such as traded stock, then our conclusions could be very different.

Let us explore a general case, where a publicly held firm is contemplating a

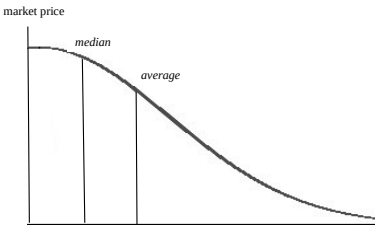
[13] There are several reasons why we should distinguish a two-tiered tender offer: (1) to affect the second stage 'short-form' merger, an exceptionally steep holding requirement of 90% shareholders in needed. See Section 253 to the DGCL. Boone et al. (2017) regard this 90% holding requirement as a super-majority voting threshold. However, a supermajority voting requirement of 90% is clearly too high and unwarranted, and therefore Delaware's 251(h) amendment to withdraw from such an exceptionally steep threshold would beneficial; (ii) All mergers under section 251(h) must be 'friendly' transactions, meaning that the board has already evaluated, negotiated, and approved them. After the preliminary screening and approval by the board, an additional requirement of a 90% super-majority vote would indeed be redundant; (iii) The 90% threshold for a 'short-form' merger in a two-tier tender offer is only a holding threshold, not a voting requirement. Therefore, in practice it was very easy to circumvent it. Almost all two-tiered tender offers included a 'top-up' option that allowed bidders to buy newly issued stock directly from the company and thus reach the 90% threshold. See Boone et al. (2017), table 2. Thus, Delaware's 251(h) amendment may only eliminate the unnecessary transaction costs associated with the 'top-up' option.

fundamental corporate change such as: to oust the incumbent management and approve a new slate of directors, to adopt a new set of bylaws, to initiate a reorganization, or perform a stock-for-stock merger with another publicly traded company. Such a proposed material change, if approved, could significantly impact the firm and its market price, Cuñat et al. (2012).

Heterogeneous shareholders, who are now faced with a proposition that can materially change the corporate, must assess and evaluate the expected value of the firm in two different states of the world: one without the change (if the offer will be rejected) and one with or after the material change (if it will be accepted). The crucial point for our discussion is that in either state of the world, the firm will remain publicly held and its shareholders will remain heterogeneous. Therefore, the distributions of valuations will be skewed to the right in both states of the world:



or



Option A: Current Company *Option B: Proposed Company*

When shareholders vote and choose between these two possibilities, how does the two skewed distributions affect their votes? We do not know. Although the valuations in both states of the world are skewed, we have no prior knowledge about the relationship between them and whether they are correlated or not (for example, we do not know if a very optimistic shareholder under option A, would be a pessimist or an optimist considering option B). When shareholders are deliberating between two heterogeneous possibilities, we cannot assume any systematic flaw or skew in their votes. Therefore, regarding voting on general corporate governance issues, reorganizations, or stock-for-stock mergers, we have yet to challenge the prevailing default rule of a simple-majority vote.

The problem of skewed voting is therefore limited to corporate governance decisions where shareholders are offered a homogeneous consideration, such as a voluntary dissolvent or any sale of the firm for cash.

7.2 Generalization

The problem of skewed voting, and the tendency of a simple majority of shareholders to overly accept underpriced bids, can be extended and generalized any circumstances of a collective decision, where a heterogeneous group of owners are contemplating to exchange their collective rights for a homogenic consideration (cash).

When prior to a collective decision, members of the group could exit the group and sell their rights for a certain (market) price, then we can expect that any pessimists have already left the group and only optimists remain. Thus, such a collective decision is susceptible to the problem of skewed voting, and the risk that the majority members of the group would approve a socially decreasing transaction, in which the asset is sold for a price lower than its worth.

Due to the problem of skewed voting, I argue that some 'end-of-life' collective decisions must not be decided solely by the collective vote of a simple majority of members, but instead must be supplemented (or replaced) by other filtering mechanisms that will be better attuned and aligned to the group's average preference.

There are several examples and possible scenarios on which the problem of skewed voting may arise. In addition to a corporate shareholders' decision to sell the company (or merge) for cash, which we have discussed above, the same phenomenon may occur in a shareholders' decision to dissolve the company. Other possible circumstances could be various partnerships or collective real estate owners, such as condominiums, who are faced with a collective decision to dissolve the partnership or sell their jointly held assets.

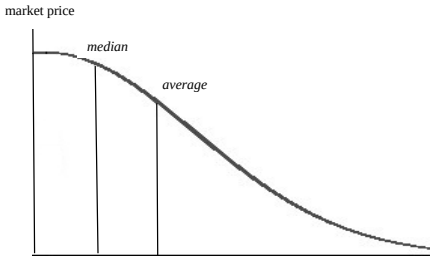
8. Extension 1: Collective Purchases

Up until now, I discussed situations where a group of *owners* of an asset are holding a vote on the question of whether to sell it or not. I will now turn to the opposite situation and discuss a case of a group of potential *buyers* who hold a vote on the question of whether to collectively purchase an asset. An example of such a case is a group of shareholders of an empty shell cooperation such as a SPAC (Special Purpose Acquisition Company) that vote on whether to approve a purchase of a new business activity by the company. The analysis in such a case is similar to the analyses above,

although the conclusions are inverted, as the group will tend to reject efficient and socially enhancing purchases.

8.1 The Theory

Prior to the collective purchase, the potential buyers are assumed to be *homogeneous*, as they collectively hold only cash. After the purchase, buyers are assumed to be *heterogeneous*, as they hold different preferences and valuations about the value of the purchased asset. We also assume that post acquisition, buyers will be able to exit and sell their rights for a market price. Therefore, as before, buyers' valuations are asymmetric and positively skewed.



Distribution of Valuations After Purchase

However, unlike the previous model for a collective sale, since they vote whether to purchase an asset, their collective decision is distorted in the opposite direction and will be *over-pessimistic* and might reject beneficial purchase offers.

To understand why, assume the buyers are offered to purchase the asset for a price between the median and the average. Since the asking price is below the average, the group as a whole will benefit for such a purchase. However, since the asking price is above the median, the majority members of the group will vote to *reject* the purchase. Buyers will over-reject beneficial purchase offers.

8.2 Examples

The problem described above, of buyers overly rejecting beneficial purchases, may arise in several corporate and collective action situations. Consider a situation where a company or a group of investors are considering a transaction where they exchange

cash (or another homogeneous valued asset) for the stock of a publicly traded company (or another heterogeneously valued asset). Another example is when a closely held company (whose shareholders may be homogeneous) is considering a merger with a publicly traded company, or when a closely held company is considering an initial public offering (IPO) that will make its shares tradable in the stock market. In all these examples, there is a risk that a simple-majority vote by shareholders will reject beneficial offers.

8.3 Possible Solutions

Note that in case of a collective purchase with a positive skew, a heightened super-majority voting rule will not solve the problem, but only worsen it. So, what are the corporate governance mechanisms available to solve such a problem of an under-acceptance of purchase offers?

One theoretical option is to lower the voting threshold needed to approve a purchase decision to less than 50% of shareholders. Such a solution may seem at first glance to be unfeasible. However, considering the current allocation of powers between shareholders and management, subjecting major purchase decisions to a shareholders' vote with a less than 50% threshold may serve as a reasonable compromise and an effective supervision tool by shareholders. Another approach could be to allow for an unequal treatment to different members of the group, such as side payments or buyouts of dissident members. A third possible approach, one that is often used in contemporary corporate law, is to avoid shareholders' voting all together on purchase decisions, and instead authorize the Board of Directors to be the sole authority in such matters with no need for prior shareholders' approval.

The question of the proper authorization process needed for material corporate purchases and whether shareholders should be entitled to a vote on this issue, is complex and unresolved. The empirical evidence in this matter is mixed and inconclusive. See Hsieh & Wang, (2008); Ehud Kamar (2011); Becht et al. (2016); Li et al., (2018); Mason et al (2018); Becht et al. (2021).

This research adds a valuable brick in this ongoing debate. I argue that when corporate purchases increase the level of shareholders' uncertainty and heterogeneity, subjecting such decisions to a shareholders' majority vote might lead to a rejection of

beneficial corporate purchases.

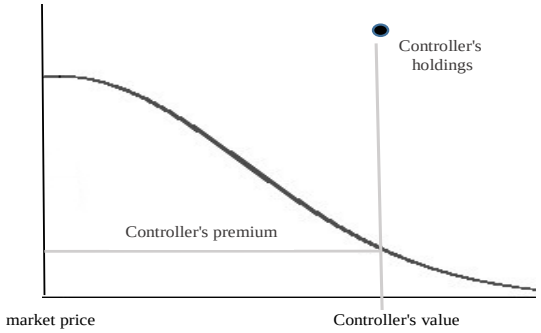
9. Extension 2: Voting in Controlled Companies

Many publicly traded companies are controlled companies (Kamonjoh, 2016). In a controlled company, a single shareholder or a coalition of several shareholders hold most of the shares and the voting rights in the company, and thus control the voting process. In a controlled company, there is a risk that the controller would force a transaction that would be beneficial only to himself but detrimental to the minority shareholders. In this section, I try to analyze how shareholder heterogeneity would take part in controlled companies.

9.1 Unconflicted Transactions

Let us first consider a situation where a controller wishes to sell the entire company to a third party, and the controller does not have any special or personal interest in the transaction. Such unconflicted transactions are usually subject to the lenient 'Business Judgment Rule' standard.[14]

In a controlled company, the distribution of shareholders' valuations can be described in the illustration below:



Distribution of Shareholders' Valuations in a Controlled Public Company

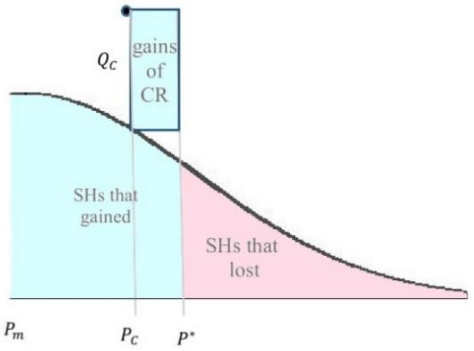
A controlled company has two types of shareholders: the public shareholders,

[14] See Delaware cases such as *In Re: John Q. Hammons Hotels Inc. Shareholder Litigation,* C.A. No. 758-CC (Jan. 14, 2011); *In re: Martha Stewart Living Omnimedia, Inc. Stockholder Litigation*, C.A. No 11202-VCS (Del. Ch, Aug. 18, 2017).

which are heterogenic and, as before, their distribution is skewed to the right; and the controller, a single shareholder with a significant holding in the company (described in the vertical grey line) that enjoys a significant positive control premium from his shares (described in the horizontal grey line). Due to the controller's significant holdings and unique premiums, the distribution of shareholder's valuations is more scattered and there is no assurance about the relation between the median and the average valuations.

The existence of a controller can mitigate the problems of skewed voting and of an overly eager simple majority approving an underpriced bid. In a controlled company, a transaction will take place only if the controller endorses and approves it, and the controller will approve a bid only if it offers him a price that exceeds his control premium, thus guaranteeing substantial gains for many minority shareholders. A controller's approval of an unconflicted transaction is a strong indication that the bid is socially desirable, and most shareholders gain from it. The higher the controller's holdings or the controller's premiums are, the greater the chances are that the controller approved a socially desirable the transaction.

Formally, let $x \sim N(P_m, \sigma^2)$ be a normal distribution function. The distribution function of current shareholders' valuation will be truncated, starting from P_m, which is the current market price. Let P_C be the value of the control shares to the controller. Let Q_C be the quantity (ratio) of shares held by the controller (and $1 - Q_C$ the ratio of shares held by the public) and let P^* be the bid price offered by the buyer.



Gains and Losses in a Sale of a Controlled Company

We assume a positive control premium $P_m < P_C$. The bidder must offer a price P^* that will exceed the controller's control premium: $P_m < P_C < P^*$. If the control premium is very high $P_m \ll P_C$, then the bid P^*must also be high $P_m \ll P^*$. Such a high bid will most likely be socially enhancing.

However, it is possible that a socially decreasing bid would be approved by a controller and a simple majority of shareholders. Two conditions must be met: First, the proposed bid needs to receive an approval of the majority, meaning that the ratio of shares held by the controller and other supporting shareholders must be greater than 0.5. Likewise, the ratio of shares held by the opposers must be smaller than 0.5.

$$\int_{P^*}^{\infty} f(x)dx < \frac{1}{2} < Q_c + \int_{P_m}^{P^*} f(x)dx$$

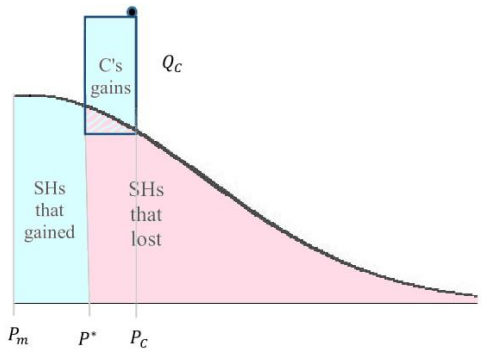
Second, the proposed deal is inefficient according to the Kaldor-Hicks criteria, meaning that the aggregated shareholders' losses are greater than shareholders' (including controller's) gains:

$$Q_c(P^* - P_c) + \int_{P_m}^{P^*} (P^* - x) \cdot f(x)dx < \int_{P^*}^{\infty} (x - P^*) \cdot f(x)dx$$

Note that in a controlled company, because we do not know the relation between the median and the average, a reverse problem cloud also occurs: a controller might tend to reject a socially enhancing and desirable bid. The formal conditions for this problem are similar to the conditions described here, with the inequality signs reversed.

9.2 Freeze-Outs and a Supermajority-of-the-Minority (SMOM)

Let us now consider a freeze-out transaction, a related party transaction (RPT) in which the controller is the bidder who forces the purchase of all remaining shares held by the public and becomes the sole owner of all the company's shares (a going-private transaction). In such a conflicted transaction, there is a greater risk that the transaction would be socially decreasing. To understand why, consider the illustration below.



Gains and Losses in a Freeze-out Transaction

In the freeze-out transaction, the controller is the bidder who unilaterally choose the proposed bid price offered to minority's shareholders - P^*. This price is now expected to be *lower* than controller's controller premium, $P^* < P_C$ (compare to unconflicted bids where $P_C < P^*$). The controller is incentivized to lower his bid price that he proposes to shareholders, and thus might force an inefficient transfer of minority shares to himself.

Corporate law offers several legal mechanisms to deal with such conflicting and potentially harmful transactions, see Subramanian (2005). Many jurisdictions require that such a transaction would be subject to an approval of a majority of the minority, a *MOM* condition. See Goshan (2003), Rock (2018). This model, however, would argue that to adequately protect a heterogeneous group of minority shareholders, a majority-of-minority vote (MOM) may not be sufficient and therefore, corporate law should consider implementing a *super-majority*-of-minority vote (SMOM).

The question of whether freeze-out transactions should be approved by a heightened *super-majority* vote of the minority (SMOM), lays upon a theoretical dilemma, of whether to include the controller's gains in our efficiency considerations.

The common approach is that freeze-out transactions should be heavily regulated, to ensure such a RPT would perform similarly to any other common commercial transaction, between a buyer (controller) and a group of sellers (minority shareholders). In such an arm's length transaction, the deal price should be negotiated by the opposing parties and then, a Pareto efficiency can be achieved.

Formally, a freeze-out transaction must not harm the minority shareholders, and their gains should be higher or equal to their losses:

$$\int_{P^*}^{\infty}(x-P^*)\cdot f(x)dx \le \int_{P_m}^{P^*}(P^*-x)\cdot f(x)dx$$

Under this efficiency condition, the same exact model for shareholder heterogeneity should apply, as if the minority shareholders are now the selling shareholders and the controller is now the outside bidder. As before, the distribution of valuations by the minority shareholders' is skewed to the right; therefore the required voting threshold should be an elevated *super-majority* vote of the minority shareholders (SMOM). Also applicable are all other alternative solutions we discussed before in section 5 above, including a monetary compensation (an appraisal right) or the negotiation by a professional agent of the minority (a Special Committee).

An entirely different approach is to view the controller as an integral part of the company and as one of its shareholders. To maximize social welfare, we should aggregate the welfare of all types of shareholders, including the controller. Under such an approach, there is no need to guarantee a Pareto efficiency, but only to ensure an overall aggregated Kaldor-Hicks efficiency. Under such an approach, it is acceptable for a forced freeze-out to financially injure the minority shareholders, as long as the controller's gains exceed shareholders' losses. Formally:

$$\int_{P^*}^{\infty}(x-P^*)\cdot f(x)dx \le Q_c(P_c-P^*)+\int_{P_m}^{P^*}(P^*-x)\cdot f(x)dx$$

Under this second view, a super majority of the minority vote (SMOM) is not required, as there is no need to align the pivotal voter to the preference of the average minority shareholders. Moreover, even the common and wildly used simple-majority-of-minority vote (MOM) may be too strict, as the aim of regulation is not to protect the minority's welfare, only to ensure an overall efficiency. Note however, that due to the controller's distorted incentives, a simple-majority vote is not sufficient to ensure an efficient transfer, and some form of regulatory protection is still required.

10.Possible Criticism

10.1 Defining Shareholder heterogeneity

Building on Miller's (1997) work, I assume that shareholders of publicly held companies are heterogeneous in their valuations of their shares. However, it is worthwhile to shortly explain what I mean by shareholder heterogeneity.

In its narrow definition, shareholder heterogeneity means that shareholders differ in their projections and expectations about the future value of their shares. For instance, when shareholders differ in their interpretations of available information (e.g. Li, Maug & Schwartz-Ziv, 2019). However, in a different broader sense, shareholder heterogeneity may also refer to difference in shareholders' preferences. Meaning, that different shareholders derive different utilities from the same shares. Consider, for example, the worth of a share to an investor, versus its worth to a manager or a controller.

Therefore, there can be many reasons why different shareholders can produce different utilities from the same shares.[15] Such reasons may be tax considerations, attitude towards risk or uncertainty, trading strategy and time horizons, human-capital investment in the firm (employee, manager), power to influence the future operation of the company (manager, director, controller), shareholder's portfolio composition, liquidity constraints, different interpretations of market information, private information, various economic dependency in the company (consumers, employees), preference towards corporate governance or various social issues ('investor ideology'), difference legal status of the holder (shares held by a fiduciary or by the state), differences in various rights attached to the shares (shares held in different jurisdictions or purchased through different stock exchanges).

To be clear, the analysis and the conclusions of this research are valid under either a narrow or broad definition of shareholder heterogeneity.

[15] See for example, Hayden and Bodie (2018), Li, Maug & Schwartz-Ziv (2019, Section 2.2.3).

10.2 Why Optimists Shareholders do not Buy all Remaining Shares?

One may argue that traditional finance theory rejects the hypothesis of shareholder heterogeneity and assumes that market participants are all homogeneous and value stock at its current market price. If an optimistic shareholder does exist, then he would simply buy all available and underpriced stock. Therefore, shareholder heterogeneity cannot really exist in the markets.

This criticism does not address the analysis of this paper but instead directs towards the underline assumption of shareholder heterogeneity. Therefore, my first response in fending this criticism is that shareholder heterogeneity cannot be easily dismissed. The notion of shareholder heterogeneity has been thoroughly researched and backed by substantial supporting empirical evidence (see review in Section 2 above). The reality of shareholder heterogeneity also congruous with our common knowledge and experience about how the capital markets work. Each transaction in the market is evidence of the difference in valuations between a seller and a buyer, and each non-unanimous corporate vote is a vivid demonstration of the different views and various preferences hold by shareholders.

Nevertheless, the question remains: why optimistic shareholders do not buy all remaining stock, as they stand to gain for such additional purchases? Several possible explanations (or holding costs) can be raised in response: One obvious explanation is the enormous financial costs that are required to purchase large amount of stock; a second explanation is the huge risks associated with an undispersed and concentrated ownership of large amounts of stock in a single firm; a third explanation may be that the continuous purchase of available stock will elevate stock prices and therefore will reduce the incentive of buy additional stock.

Whatever the reasons for this puzzle may be, any assumption of shareholder heterogeneity must also assume some sort of limitations or costs associated with accumulating growing amounts of shares. The reality of the stock markets is one of wide dispersion and of shares held by a large number of shareholders, and not one of a firm exclusively held by a single shareholder.

10.3 Shareholders' Distribution Function

The main argument explored in this research is that in publicly held companies, the distribution function of shareholders' valuations is skewed to the right, and shareholders' average valuation is above the median. This assertion is, of course, merely a general statement or a default assumption and should not be mistaken as a strict and absolute rule.

Although it safe to assume that all distribution functions are truncated from the left by the market price, one may argue that not every truncated distribution function is necessarily skewed to the right. In some truncated distribution functions, the average is not higher than the median. Consider, for example, a uniform distribution or an increasing function. However, it seems very unlikely that shareholders' valuations are distributed is such ways. Consider the empirical data mentioned in Section 2 above, which finds evidence that shareholders' demand function curves downwards. The traded market price is a strong reference point for shareholders. Therefore, it will be very surprising if only few shareholders cling to it and more and more shareholders drastically deviate from it.

However, I do not assert that distribution functions in *all* firms must be skewed to the right, but only that such a default assumption is more plausible than the current assumption of symmetric distribution functions. This model assumes the shareholders' valuations are initially distributed normally, and this seems like a more reasonable starting point than a uniform or a U-shape distribution. To sum up, there are more theoretical reasons and empirical data to assume that shareholders' distribution is asymmetric rather than symmetric, and that skewed voting is a real problem.

10.4 Should Minority Shareholders Decide?

According to the model above, some major corporate decisions should be decided by a super-majority vote. However, in such super-majority decisions, the minority of shareholders will have a de-facto veto power over the majority's will. Allegedly, super-majority voting contradicts the concept of shareholders' democracy, and therefore corporations should stick to the prevailing simple-majority voting rule.

In response, I argue that there is nothing sacred about the simple-majority voting rule. It is merely a default rule, justified mainly because of the lack of any information about the distribution of voters' preferences. If, however, we have a strong reason to believe that voters' preferences are systematically skewed to a certain direction, then the voting threshold should be shifted accordingly.

Super-majority voting does not contradict with shareholders' democracy, but in fact, is consistent with our prior intuition and professional experience. As mentioned in Section 5.1 above, many states' laws and corporations' bylaws choose to adopt super-majority voting rules in certain voting issues. Consider even the example of the super-majority requirement needed for constitutional amendment, which can be explained on similar grounds: when a collective decision by a simple majority has the potential to cause great harm to the minority. Also note that the need for a super-majority vote only holds in decisions where a heterogeneous group trades their rights for a homogeneous value (cash), mainly meaning 'end-of-life' decisions such as the sale of the company or its dissolution (see Section 7.2 below). Subjecting 'end-of-life' collective decisions to a supermajority vote seems to concur with our prior intuition.

10.5 Should Over-Optimistic Shareholders Decide?

An obvious criticism is the intuitive wonder of why *over*-optimistic shareholders should receive substantial and even decisive influence over corporate decisions. To put this argument in other words, most investors adhere to the market price as the 'normal' or the most acceptable value of a stock. Shareholders who differ from the market norm and value their shares above market price should be considered as exceptional or abnormal (for now, abnormal but not mistaken). This research argues that corporate resolutions and the fate of the company should be determined by a decision of a small number of *abnormal* optimistic shareholders. However, one can argue that the pivotal shareholder should be a 'normal' shareholder who holds the most acceptable market valuation, and not an unconventional and abnormal optimist shareholder.

More than a few answers can be given to such an argument: *First*, the basic axiom for our discussion is that social welfare is derived from individuals' subjective preferences, and that subjective preferences are considered worthy even if they deviate from a certain common or popular preference. There is no good theory to suggest why

we should ignore subjective valuations of heterogeneous well-informed shareholders.

Second, even under the current simple-majority default rule, the pivotal shareholder who decides the fate of the vote is not the marginal shareholder who holds the market valuation, but the *median* shareholder who also holds a far above the market valuation, see Bernhardt, Liu & Marquez (2018). We can observe the significant difference between the market price and the deal price by the substantial premium bidders must offer and pay shareholders in corporate transactions. See Ruback & Jensen (1983) or Jarrell, Brickle & Netter (1988). So even under the current default rule of a simple-majority voting, collective corporate decisions are not determined by the market valuation but by a higher median valuation, as should be. Note, that if we were to take this criticism to its extreme and wish that the pivotal voter would be a marginal shareholder that holds a market valuation, then we should lower the voting threshold to be less than 50%, clearly an undesirable outcome.

10.6 Should Erroneous Shareholders Decide?

A follow-up criticism might argue that many of the shareholders who hold high valuations over the market price hold no special or unique set of preferences about their shares. Instead, these shareholders are plainly wrong and simply mistaken in their over-optimistic valuations. If indeed these shareholders are misevaluating the correct value of their shares, then aggregating these erroneous preferences might lead to the wrong voting outcomes.

In response, we should first note that there is no solid theory or empirical evidence suggesting a systematic failure by shareholders to accurately evaluate the value of their own stock. Second, it is unlikely that *all* 'high-valuating' shareholders are mistaken. A more plausible assumption is that although some shareholders might be mistaken about their optimistic valuation of the firm, many other shareholders are truly heterogeneous and hold genuine high valuations on their stock.

If we could identify in advance which of the shareholders are mistaken in their valuations, then maybe the proper response should be to withhold these mistaken shareholders from voting or require that a transaction would be subject to a majority of the 'unmistaken' shareholders (similarly to the approval process of self-interested

transactions, where we often require an approval by a majority of the unconflicted shareholders, the majority of the minority). However, as the remaining shareholders are heterogeneous and hold 'true' valuations, a super-majority vote would still be required for the 'unmistaken' majority (similarly to the analyses in Section 9.2 above).

10.7 Should Outside Investors Decide?

A critic from a different direction can argue that when a court assesses the value of a firm or its shares, the court should base this value according to the average value by all market participants. The intuition here is that the perceived value by all market participants would produce a more accurate prediction (consider, for example, the Condorcet's jury theorem or the Efficient Market Hypothesis). When assessing the value of the shares, the court should not limit itself to optimistic valuations held only by current shareholders, but instead should also consider valuations from various non-shareholders' investors. The average value of all market participants, which essentially is the currently traded market price, would be a more accurate measurement of the value of the firm.

In response, note that such criticism has no merit regarding *voting* and the main conclusions of this research. We consider corporate voting as aimed to benefit shareholders of the corporation, and not for the benefit of the market. Therefore, we count votes and aggregate preferences of current shareholders and disregard votes and preferences of other investors or stakeholders. Because corporate voting is strictly limited to shareholders and shareholders' voting, preferences and valuations by current shareholders would be skewed to the right by the overly optimistic shareholders.

However, such criticism may have some merit considering *compensation* of shareholders. Here we reach a theoretical crossroads regarding the purpose of compensation: Should courts compensate shareholders according to the acceptable ('objective') market value of the shares taken, or should courts award damages according to the injured shareholders' own private ('subjective') valuations? A comprehensive discussion on this topic exceeds the limits of this paper. I will only note that I assumed here the later. Meaning, that in a forced taking of corporate shares (such as a merger or a freeze-out) shareholders' compensation should be aimed to fully compensate the shareholders themselves for the subjective harm caused to them. Such

level of compensation would guarantee that any approved transfer would be Kaldor-Hicks-efficient. This approach also disregards non-shareholders' valuations.

10.8 Miller's (1977) Theory

The market mechanism that causes various shareholders to be over optimistic and hold above-the-market valuations was first described by Miller (1977). Miller theorized that a market with heterogeneous shareholders and short-sale constraints would result in an elevated market prices that reflect the valuations of optimist shareholders. Miller's theory has been subsequently supported by empirical evidence exploring the connection between shareholder heterogeneity and short-sale constraints to stock prices. See for example, Chen et al., (2002) and Boehme et al. (2006).

This model builds on Miller's work, but differs from it in two aspects: First, Miller focused on how shareholder heterogeneity affects *market price*, while this model focuses on how shareholder heterogeneity affects the *shape of the distribution curve*, the divergence between average and median valuations, and various corporate governance issues that arise from this divergence such as: super-majority voting, the division of powers between shareholders and managers, and appraisal rights.

Second, Miller's (1979) theory is conditioned on the existence of short-sales constraints, while this model is apparently not. According to Miller's (1979) theory, shareholder heterogeneity elevates market prices while short selling lowers it. Short selling increases the supply of stock by allowing for additional shares to be sold in the market (more than 100%). These additional 'fictional' shares allow for more pessimistic investors to enter the market and become shareholders, thus lowering the market price, see Miller (1979, p. 1160 -1162). In other words, shareholder heterogeneity generates a positive market pressure that elevates stock prices. This upward pressure could be countered or mitigated by an opposite market pressure downwards, generated by the short sales. Miller theory therefore requires heterogeneity *and* short-sale constraints.

10.9 Effects of Short Selling

Building on Miller's (1977) theory, could short sales also affect this model, and could short sales correct the skew in shareholders' valuations? Without empirical data we cannot present a conclusive answer. However, there are several strong reasons to assume that shareholders' asymmetry will persist, even whenever shorts sales are wildly available and unconstraint.

For start, short sales are, by their nature, more regulated and limited than the common 'long' sale. Short sales are often regulated and restricted by law, see Avgouleas (2011), Howell (2018). The purchase of shorts also involves exceptionally high transaction costs, commissions (rebate rates), and the requirement to supply financial guaranties. Therefore, in practice, a market for shorts is not widely available for any stock, and even when short selling is wildly available, trading volumes in shorts are often lower compared to trading volumes in regular trade in 'long' stock.

But even if shorts sales were to be free from any regulation, high costs, and wildly available, even then the assumption of asymmetry in shareholders' valuations is doubtful. First, even whenever short sales are wildly available, shareholders' valuations are still strictly bounded from the left (due to the limited liability principal) and are not bounded from the right (as shares' values may rise to infinity). Note that although short-sale investors are exposed to infinite negative risk (as the price of stock they committed to buy could rise infinitely), these investors are *not* shareholders. They do not vote nor are entitled to receive any compensation during the sale of the company. This second point is crucial to our discussion, as shorts sales indeed increase the *supply of stock*, but they do not increase the *supply of votes*, as both the lender of stock and the short-sale investor are unable to vote.[16]

The short-sale transaction involves three participants: the short-sale investor, the lender, and the new buyer: (i) *The short seller* is a pessimistic investor that holds a low valuation on the company's stock. The short seller sells the stock without buying it, and in no time during the short-sale transaction does he become a shareholder of the company. Short sellers are never shareholders; therefore they are not entitled to vote,

[16] Although, there is some evidence of illegitimate over-voting that is caused by short sales, see Hu & Black (2006, p. 897- 898). But it seems that this technical problem of over-voting is diminishing. See Securities Transfer Association (2018), stating that: "the over-voting problems reported in years past have significantly diminished."

and their pessimistic valuations are not reflected in shareholders' valuations; (ii) *The lender of the stock* is a current shareholder who agrees (expressively or implicitly) to lend his shares for the purpose of the short transaction. During this lending period, the lender is not entitled to vote, as his shares are no longer in his possession. Formally, the lender is also irrelevant to the shareholders' voting process; (iii) *The new buyer* is the investor who bought the lent stock, and he is now the formal holder of the shares and owns their voting rights. There is no apparent reason to assume that the new buyer would vote any differently from the original shareholder (the lender). Furthermore, in view of a forthcoming significant vote, lenders can retain the right to recall their shares and reclaim their voting rights. Their broker will then need to buy back some new shares from the market, and these buybacks will discard some marginal, pessimistic, and 'low-valuating' shareholders, bringing us back to the optimist 'high-valuating' shareholders. To sum up, the availability of shorts sales is not expected to have any major effect on the distribution of (voting) shareholders' valuations.

11.Concluding Remarks

Extensive research has shown that shareholders of publicly traded companies are heterogeneous and hold different preferences and valuations on the value of their shares. Miller (1977) has shown that the tendency of pessimistic shareholders to sell their shares and exit the company will result in elevated share prices. In this research, I concentrate on the shape of the distribution function of current shareholders' valuations, and its impact on various corporate governance issues. I describe how shareholder heterogeneity can cause a positive skew in shareholders' distribution function and how this skew entails significant corporate governance implications.

I discuss three distinct valuations held by various shareholders: (i) the marginal value, which is the currently traded market price and is the lowest share valuation held by shareholders; (ii) the median value, which is the share value held by the pivotal shareholder in a simple-majority vote; and (iii) the average value, which represent the aggregated value of all shareholders, and therefore is the pivotal share value for a socially enhancing transfer of the company. In asymmetric distribution of valuations, we can expect the average to coincide with the median. However, in a publicly traded company, the distribution of valuations is asymmetric and skewed to the right, and

therefore the average valuation is higher than the median.

My main argument concerns shareholders' voting. A typical shareholders' vote is decided by a simple majority of shareholders and based on the preferences of the median voter. However, a socially enhancing transfer of the company must be based on the average share value, which is typically higher than the median. Therefore, a simple majority of shareholders might vote to approve or agree to tender in an underpriced bid. I argue that shareholders' collective decisions to sell the company for cash should not be based solely on a simple majority vote, but instead should be re-adjusted to better represent shareholders' average preference. I discussed several possible alternatives to adjust shareholders' skewed voting, such as a super-majority vote, an appraisal remedy adjusted to the average value, and a price negotiated by the Board of Directors (hence, a board's veto right). Lastly, I discuss which types of voting decisions might be distorted due to shareholder heterogeneity.

Considering shareholders' compensation, I also argue that shareholders' compensation should be adjusted to represent the average share valuation held by shareholders, and such an average value is greater than the market price (marginal value) or shareholders' approved price (median value).

This research sheds light on the relationship between shareholder heterogeneity and corporate voting and various other corporate governance issues. It is only a first step, a toehold, in this line of research. Further theoretical and empirical work can be done, among other things, to explore the degree of shareholder heterogeneity, to explore various methods to assess shareholders' median and average valuations, to suggest what super-majority threshold should be used to best represent shareholders' average preferences. Further research is also required to investigate the interrelationship between voting and trading (recently, Levit et al. 2021) and to other types and circumstances in which collective decisions suffer from skewed voting.

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13. Appendix: Formal Proof

We would like to prove that the median is smaller than the mean in a truncated probability density function (PDF). The only assumption made here is that the PDF was a normal distribution before it was truncated by the market price.[17] As the PDF is a single peak function, the truncation (the market price) can occur either to the right or to the left of the mode, and I analyze both possibilities.

13.1 Truncation Right of the Mode: Descending PDF

In the first part of the proof, I assume the PDF is truncated at to the right of the mode; therefore the PDF is monotonically decreasing. In the corporate setting, this assumption seems very reasonable: most shareholders tend to value their shares at or close to the market price. All other shareholders are more optimistic and value their shares above market price, but their numbers decrease as they move further away from the market price.

The median X_{med} is the value at which the integral of the PDF is just half:

$$X_{med} = \int_{X_0}^{X_{med}} P(x)dx = 0.5$$

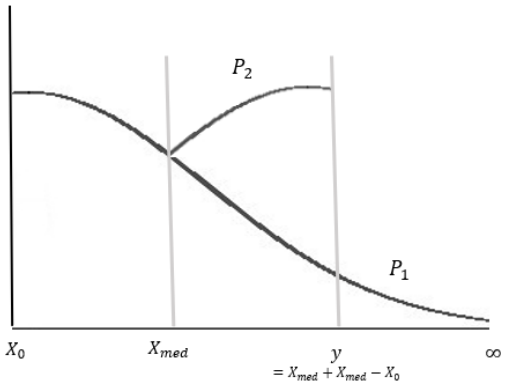
[17] The assumption of a normal distribution function can be eased to any symmetric convex single-peaked function. Alternatively, it is also possible (and even reasonable) to assume that the PDF is monotonically descending, as I discuss in sub-section (i) below.

The mean of the PDF is defined by:

$$X_{mean} = \int_{X_0}^{\infty} P(x)xdx$$

(I use the sign ∞ to represent the highest defined x in the PDF, which is not necessarily infinity).

We will now draw another PDF around the median. The left half of both PDF's is identical. The right half of the second PDF will be a mirror image of the left half, so the second PDF is symmetrical around the median.



We define $P_1(x)$ as the right half of the original PDF and define $P_2(x)$ as the right half of the second PDF function. The function $P_2(x)$ ends at $X_{med} + X_{med} - X_0$. For convenience, let us define: $y = X_{med} + X_{med} - X_0$. (Note that $y < \infty$. This is true because we assume that the original PDF is generally descending, therefore, its mirror P_2 will be generally increasing, and thus P_2 must be shorter than the P_1).

Since these two functions are on the right hand-side of the median and since they are both half of a PDF, we obtain that both P_1 and P_2 equal 0.5:

$$\int_{X_{med}}^{y} P_2(x)dx = \int_{X_{med}}^{\infty} P_1(x)dx = 0.5 \quad (1)$$

The right-hand side of eq. (1), $P_1(x)$, can be divided into two parts:

$$\int_{X_{med}}^{\infty} P_1(x)dx = \int_{X_{med}}^{y} P_1(x)dx + \int_{y}^{\infty} P_1(x)dx = 0.5 \quad (2)$$

The left-hand side of eq. (1), $P_2(x)$, can be rewritten as:

$$\int_{X_{med}}^{y} P_2(x)dx = \int_{X_{med}}^{y} [P_2(x) - P_1(x) + P_1(x)]dx \quad (3)$$

This, again, can be rewritten as:

$$\int_{X_{med}}^{y} [P_2(x) - P_1(x)]dx = \int_{X_{med}}^{y} P_2(x)dx - \int_{X_{med}}^{y} P_1(x)dx \quad (4)$$

From eq. (1) and (4) we obtain that:

$$\int_{X_{med}}^{y} [P_2(x) - P_1(x)]dx = \int_{X_{med}}^{\infty} P_1(x)dx - \int_{X_{med}}^{y} P_1(x)dx \quad (6)$$

Which can be written as:

$$\int_{X_{med}}^{y} [P_2(x) - P_1(x)]dx \;=\; \int_{y}^{\infty} P_1(x)dx \quad (7)$$

Multiplying both sides by y, will not change the equation:

$$\int_{X_{med}}^{y} y[P_2(x) - P_1(x)]dx \;=\; \int_{y}^{\infty} yP_1(x)dx \quad (8)$$

Since for all values of x on the left-hand side $x < y$, and since for all values of x on the right-hand side $y < x$, then we obtain that:

$$\int_{X_{med}}^{y} x[P_2(x) - P_1(x)]dx < \int_{y}^{\infty} xP_1(x)dx \quad (9)$$

Let us go back now to calculate the mean of the original PDF:

$$X_{mean} \;=\; \int_{X_0}^{\infty} P(x)xdx \;=\; \int_{X_0}^{X_{med}} P(x)xdx \;+\; \int_{X_{med}}^{\infty} P_1(x)xdx \quad (10)$$

The right-hand side can be rewritten as:

$$\int_{X_0}^{X_{med}} P(x)xdx + \int_{X_{med}}^{y} P_1(x)xdx \; + \int_{y}^{\infty} P_1(x)xdx \; = X_{mean} \quad (11)$$

To the inequality in eq. (8) we can add the variables in eq. (10), then we obtain:

$$\int_{X_0}^{X_{med}} P(x)xdx + \int_{X_{med}}^{y} P_1(x)xdx \; + \int_{y}^{\infty} P_1(x)xdx \; + \int_{X_{med}}^{y} x[P_2(x) - P_1(x)]dx < X_{mean} + \int_{y}^{\infty} xP_1(x)dx \quad (12)$$

Which can be rewritten as:

$$\int_{X_0}^{X_{med}} P(x)xdx + \int_{X_{med}}^{y} xP_2(x)dx < X_{mean} \quad (13)$$

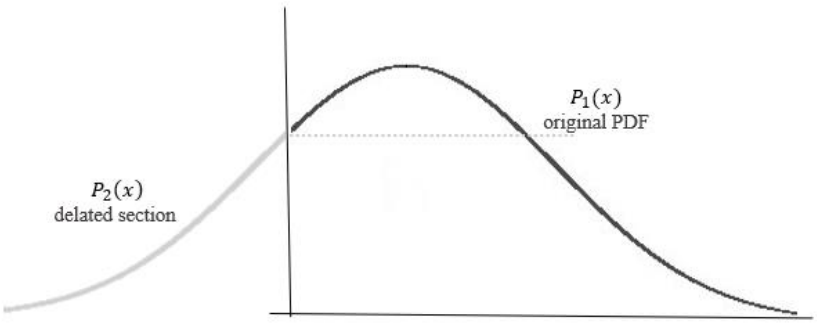
Not that the left-hand side of the inequality (13) describes the mean of the second PDF. This second PDF is symmetric around X_{med}. Therefore, its median and mean quals X_{med}. We finally obtain that:

$$X_{med} < X_{mean} \quad (14)$$

This finalized the first part of our proof.

13.2 Truncation Left of the Mode

Let us now examine the other alternative, where the market price truncates the PDF to the left of the mode.



To prove that the mean is higher the median, we will draw back the truncated left side of the original PDF, left of the market price. Let us define $P_1(x)$ as the original PDF of current shareholders and let $P_2(x)$ be the newly drawn function.

The original $P_1(x)$ plus the newly added $P_2(x)$ represent the entire PDF before the truncation. According to our starting assumption, this combined PDF is normally distributed, hence in the combined $P_1(x) + P_2(x)$ the mean coincides with the median.

Let us now consider the newly added $P_2(x)$. This is a left part of a normal distribution, which ends left of the mode. $P_2(x)$ monotonically *rise*s. Using the previous proof above, we can say that in $P_2(x)$ the mean must be *smaller* than the median.

Let us now combine these two conclusions: in the two sections $P_1(x) + P_2(x)$ combined, the mean coincides with the median; However, in only the left section $P_2(x)$ the mean is smaller than the median. Therefore, it must be that in remaining section $P_1(x)$, the mean is higher than the median. This concludes our proof.